SPECTRAL PROPERTIES OF ANDREEV REFLECTION FROM QUANTUM TURBULENCE IN ^{3}HE-B. WHAT DO THEY TELL ABOUT TURBULENT FLUCTUATIONS?

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A superfluid in the zero temperature limit has no viscosity. A superfluid Fermi liquid ^{3}He-B is formed by the condensation into the groundstate of the Cooper pairs of ^{3}He atoms. The condensate pairs behave collectively and are described by a coherent macroscopic wavefunction. The superfluid velocity is proportional to the gradient of phase of the wavefunction, $\boldsymbol{\nabla}\theta$, so that the superflow is irrotational, $\boldsymbol{\nabla}\times\mathbf{v}=\mathbf{0}$, but can support the line defects with $\boldsymbol{\nabla}\times\mathbf{v}\neq\mathbf{0}$. These are quantum vortices; around each of them the phase of the wavefunction changes by 2π which gives rise to the irrotational circulating flow. Each vortex carries a single quantum of circulation, $\kappa=\pi\hbar/m_3$, where m_3 is the mass of a ^{3}He atom.

Instabilities and reconnections of quantized vortices result in the formation of the vortex tangle which exhibit complex dynamics as each vortex line moves with the collective velocity field of all other vortices. The resulting flow is known as quantum turbulence. The property that characterizes the intensity of quantum tubulence is the vortex line density, L (m^{-2}), that is the total length of vortex lines per unit volume. In the low temperature regime, $T\leq 300\,\mu\mathrm{K}$ thermal excitations no longer form the normal fluid component, but the few remaining excitations form a ballistic gas which has no influence on vortex dynamics. The experimental technique developed to measure quantum turbulence in ^{3}He-B in the zero temperature limit utilises the Andreev reflection of ballistic quasiparticle excitations from the superflow [1].

Andreev reflection arises in a Fermi superfluid as follows. The energy-momentum dispersion curve $E(\mathbf{p})$ for excitations has a minimum at the Fermi momentum p_F, corresponding to the Cooper pair binding energy Δ. Quasiparticle excitations have $p>p_F$ whereas quasiholes have $p<p_F$. On moving from one side of the minimum to the other, the excitation group velocity reverses: quasiholes and quasiparticles with similar momenta travel in opposite directions. In the reference frame of a superfluid the dispersion curve tilts to become $E(\mathbf{p})+\mathbf{p}\cdot\mathbf{v}$. Quasiparticles which move into a region were the superfluid is flowing along their momentum direction will experience a potential barrier. Quasiparticles with insufficient energy are reflected as quasiholes which almost exactly retrace the path of the incoming quasiparticles [2].

The superflow field $\mathbf{v}(\mathbf{r},\,t)$ and the dynamics of the vortex tangle are simulated by the coupled equations

$$\mathbf{v}(\mathbf{r},\,t)=-\frac{\kappa}{4\pi}\oint_{\mathcal{L}}\frac{\mathbf{r}-\mathbf{s}}{|\mathbf{r}-\mathbf{s}|^3}\times d\mathbf{s}\,,\qquad \frac{d\mathbf{s}}{dt}=\mathbf{v}(\mathbf{s},\,t)\,, \tag{1}$$

where the Biot-Savart integral extends over the entire vortex configuration, $\mathcal{L}$, and $\mathbf{s}=\mathbf{s}(t)$ identifies a point on the vortex line. Calculations, based on the tree-method [3], are performed in a cubic box of size $D=1\,\mathrm{mm}$ using periodic boundary conditions. We simulate the evolution of an isotropic vortex tangle driven by injections of vortex loops at alternating faces of the computational cube. Injecting rings of the radius $240\,\mu\mathrm{m}$ with frequency $22\,\mathrm{Hz}$, we generate the tangle with an average line density $\langle L\rangle=9.7\times10^7\,\mathrm{m}^{-2}$ which corresponds to the mean intervortex distance $\ell\approx\langle L\rangle^{-1/2}=102\,\mu\mathrm{m}$. At large scales the energy spectrum is consistent with the Kolmogorov spectrum observed in ordinary turbulence, and for large wavenumbers with the k^{-1} behavior of individual vortex lines.

We analyze the Andreev reflection of quasiparticles incident on the vortex tangle in the fixed, x-direction. The initial positions of quasiparticles are in the $(y,\,z)$-plane. The incident flux of quasiparticles can be written as [1]

$$\langle nv_g\rangle^i=\int_\Delta^\infty g(E)f(E)v_g(E)\,dE\,, \tag{2}$$

where $g(E)$ is the density of states, $f(E)$ is the Fermi distribution, and $v_g(E)$ is the group velocity of excitations. The transmitted flux, $\langle nv_g\rangle^t$ is calculated by integral (2) in which the lower limit has been replaced by $\Delta+\max(\mathbf{p}\cdot\mathbf{v})$ calculated along the quasiparticle's rectilinear trajectory [that is for the fixed $(y,\,z)$], where the velocity $\mathbf{v}$ is found from the solution of Eqs. (1). The reflected flux is $\langle nv_g\rangle^r=\langle nv_g\rangle^i-\langle nv_g\rangle^t$. The total Andreev reflection, $f_R(t)$ is the sum of Andreev reflections, $\langle nv_g\rangle^r/\langle nv_g\rangle^i$ for all positions of the $(y,\,z)$-plane.

We monitor a statistically steady state of turbulent tangle with the time-average line density $\langle L\rangle=9.7\times10^7\,\mathrm{m}^{-2}$, for which we found the time-average total Andreev reflection $\langle f_R\rangle=0.37$. We calculate the spectral characteristics of fluctuations of the total reflection, $\delta f_R(t)=f_R(t)-\langle f_R\rangle$ and of the line density, $\delta L(t)=L(t)-\langle L\rangle$, and compare these with experimental measurements of the Andreev reflection from quantum turbulence generated by a grid.

Experimental studies of the Andreev scattering were performed for quantized vortices generated by the vibrating grid, as shown in Fig. 1. Fraction of the Andreev reflection of thermal quasiparticles was obtained directly from measurements of the thermal damping on each of the three vibrating wires shown in Fig. 1 (see Refs. [1, 4] for details).

Figure 2 (left) compares the power spectral density (PSD) of the Andreev reflection obtained from simulations with that found from the experimental data. The experimental data are shown for a fully developed tangle (grid velocity

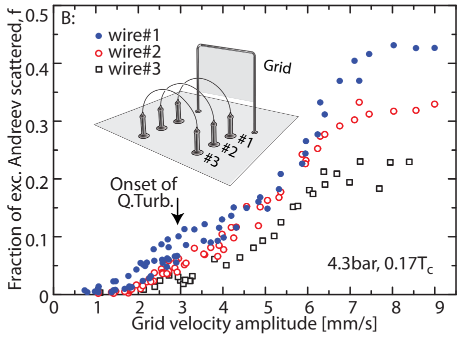

Figure 1. Experimental measurements of the fraction of Andreev reflection from quantized vortices generated by the grid.

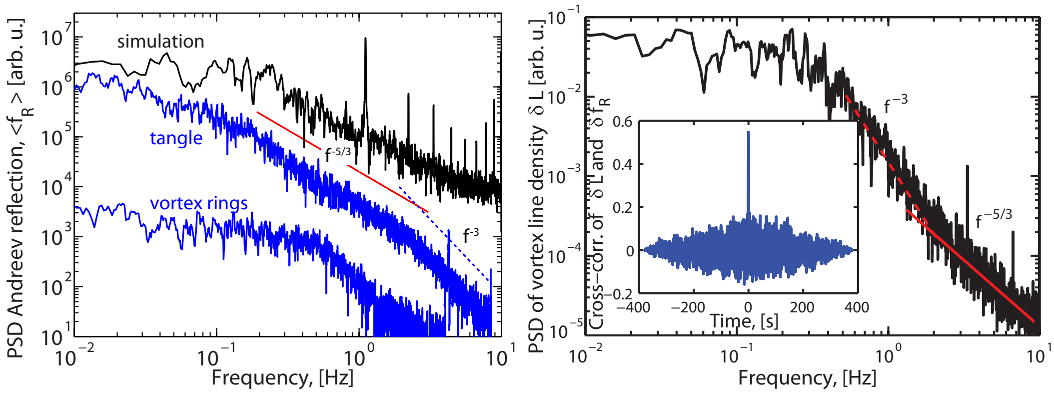

Figure 2. Left: power spectral density (PSD) of the Andreev reflection for simulations (top) and experimental observations (middle and bottom). Right: simulated PSD of the line density; inset: cross-correlation between Andreev reflection and the line density.

$6.3\,\mathrm{mm\,s^{-1}}$), and for ballistic vortex rings ($1.9\,\mathrm{mm\,s^{-1}}$). For medium frequencies f (Hz) the simulated spectral density exhibits $f^{-5/3}$ scaling in full agreement with experimental observations. For high frequencies the experimental data show steeper, f^{-3} scaling which is absent from the numerical spectrum; this scaling corresponds to the Andreev reflection from the flow at length scales smaller than the intervortex distance, $\ell \approx \langle L\rangle^{-1/2} \approx 102\,\mu\mathrm{m}$. Indeed, since at the grid velocity $6.3\,\mathrm{mm\,s^{-1}}$ the tangle propagates with the mean velocity $0.3 - 0.4\,\mathrm{mm\,s^{-1}}$ [1], then using the Taylor frozen hypothesis we find that the crossover between the two scaling regimes should be at the lengthscale 100 – 200 μm.

Unlike the simulated PSD of Andreev reflection, shown in Fig. 2 (right) the simulated frequency spectrum of fluctuations of the vortex line density has a signature of the intervortex spacing at $f \approx 20\,\mathrm{Hz}$. However, in contrast with the simulated PSD of Andreev reflection, this spectrum apparently conforms to the -3 power law for medium frequencies ($f \leq 20\,\mathrm{Hz}$). The coincidence of the $-5/3$ scalings for the two simulated spectral densities at higher frequencies is probably misleading: for $f > 20\,\mathrm{Hz}$ the experimentally observed spectral density of Andreev reflection shows a different, f^{-3} scaling. The difference in behavior of the two spectral densities seems to invalidate the earlier assumption [4] that the fluctuations of Andreev-reflected signal can be interpreted as fluctuations of the vortex line density. One of the reasons for this difference might be that, unlike fluctuations of the line density, the Andreev reflection is very sensitive to the large scale flows (caused, in particular, by the polarisation of the vortex lines [5]).

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