TURBULENT DRAG REDUCTION BY HYDROPHOBIC SURFACES WITH SHEAR-DEPENDENT SLIP LENGTH

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Abstract The stabilisation of a parabolic equilibrium profile in a three-dimensional (3D) turbulent channel flow for an incompressible fluid is addressed with the objective of achieving drag reduction. The formulation of this problem stems from Balogh's work [1] where Lyapunov stability analysis was used to devise prototype feedback laws and prove global stability of the solutions. This treatment only considers the controller as a mathematical artefact, but it can actually be linked to physical control strategies modelling hydrophobic surfaces and porous media. In the former, only linear slip velocity boundary conditions (BC) were considered [8]. However, experiments [2] have suggested that the slip length may be shear-dependent. Motivated by these, the effect on drag reduction of a shear-dependent slip length surface is examined in the present study using Direct Numerical Simulations (DNS) at $Re_{\tau_0} = u_{\tau_0}\delta/\nu \simeq 180$. δ is the channel half height, u_{τ_0} the wall-shear velocity for regular no-slip walls channel and ν the kinematic viscosity. The theoretical analysis in [5], is extended to this new model. The proposed formulation shows that the skin-friction coefficient can be reduced by tuning the parameters in the shear-dependent slip length model. The results, which verified by DNS simulations, show that by taking a slip length value based on a constant slip model [8] and combining it within a shear-dependent model, up to 50% drag reduction can be obtained. The effect of control is further assessed by formulating the Fukagata identity [4] with general boundaries; the weighted Reynolds shear-stress for each quadrant shows an enhanced reduction in the sweep/ejection events compared to the constant slip model.

INTRODUCTION

The advent of micro/nano-engineered materials to alter boundaries in a flow, raises the validity of the no-slip BC. Modelling finite slip length has paved the way for devising drag reduction techniques based on surfaces called superhydrophobic (SH) obtained by combining geometrical and chemical interface properties. Small-scale systems design, makes such surfaces promising candidates for various industrial applications involving aqueous flows. The early DNS in [8], enforced BC through an effective slip length using the following form: $f_s = l_s \frac{\partial f}{\partial y}\big|_{\text{wall}}$, where f_s is the slip velocity for the inquired direction, and l_s the constant slip length. Experiments tend to suggest that using SH surfaces may achieve drag reduction in Reynolds number ranges relevant for marine applications [5]. This brings about considering a way to increase the slip length for achieving higher drag reduction rates. In the present study, a shear-dependent slip length model is examined. In channel flow control problems, it is not always rigorously proven how the BC are devised. It was shown in [1] for a two-dimensional (2D) channel flow, how these BC originate from a Lyapunov stability analysis. The latter analysis is presently extended to the shear-dependent slip length model in a 3D channel flow. Also, the formulation by [5] is extended to the shear-dependent slip length model and corroborated by DNS simulations using Incompact3d code [7].

THEORETICAL ANALYSIS AND RESULTS

To design control laws applying to the flow boundary, partial differential equations (PDE) are analysed within vector spaces inducing norms related to physical quantities e.g. the energy of the system also called a Lyapunov function. Presently, the objective is to stabilise the flow in a turbulent channel around a specified equilibrium. The analysis carried out for a 3D channel flow is based on a previous work describing the stabilisation of a 2D turbulent channel flow [1]. The mathematical formulation for the BC can readily be linked to SH surfaces and is further extended to the shear-dependent slip length model. The Navier-Stokes equations with homogeneous directions in x and z are used. The energy of the system is expressed through the L^2 norm of the velocity vector field: $E(\mathbf{w}) = \|\mathbf{w}\|^2_{L^2}$. The formulation devised in [1] for a 2D channel used wall-tangential distributed actuation, relating linearly the shear and velocity at the walls. The latter should not be considered as a simple mathematical artefact: in fact it physically characterises hydrophobic surfaces. For the 3D derivation, the stability analysis readily relates to the three cases considered in [8]. In the constant slip model, $\forall t > 0$ the BC are $u(x, \mp 1, z) = \pm k u_y(x, \mp 1, z)$ and $w(x, \mp 1, z) = \pm k w_y(x, \mp 1, z)$. By imposing this type of BC with $k = a u_y + b$, global stability is proven in L^2 norm as the time derivative of $E(\mathbf{w})$ is computed and upper-bounded:

$$\dot{E}(\mathbf{w}) \leq -\frac{1}{2}\alpha E(\mathbf{w}) - \frac{2}{Re_p}\left[\frac{2}{b\left(1+\sqrt{1+\frac{4a}{b^2}}\right)} - 1\right]\int_0^{L_z}\int_0^{L_x}\left[u^2(x,-1,z) + w^2(x,-1,z)\right]\mathrm{d}x\,\mathrm{d}z$$
$$-\frac{2}{Re_p}\frac{2}{b\left(1+\sqrt{1+\frac{4a}{b^2}}\right)}\int_0^{L_z}\int_0^{L_x}\left[u^2(x,1,z) + w^2(x,1,z)\right]\mathrm{d}x\,\mathrm{d}z \qquad (1)$$

L_x and L_z are the channel dimensions in the streamwise and spanwise directions respectively; u and w being the perturbed velocity components in these directions. For $\alpha > 0$, $E(\mathbf{w})$ decays exponentially with time in the uncontrolled case ($u(x, \pm 1, z, t) = w(x, \pm 1, z, t) = 0$). A stringent condition follows for Re_p as $\alpha = (-4 + 1/Re_p)$. This mathematical limitation also appearing in [1], doesn't prevent the control to be effective for higher Re_p values in numerical simulations. Stability enhancement requires $a \leq (1 - b)$. The previous formulation highlights the role of boundary terms and provides a starting point to design controllers for enhancing global stability. The derivation for the shear-dependent slip length is

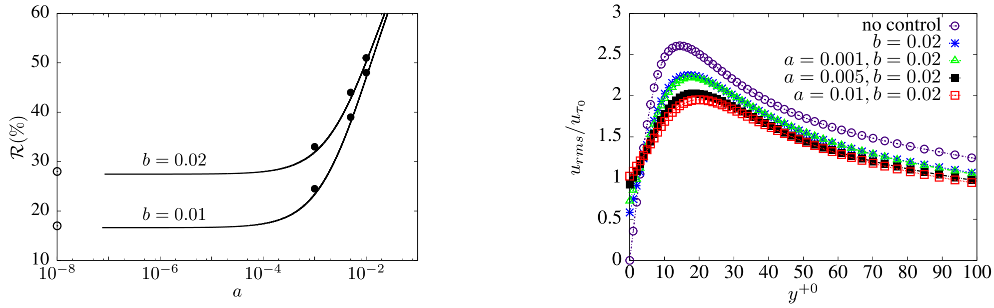

Figure 1. Left: Theoretical prediction from Eq.(2) compared with DNS. Right: r.m.s. of the streamwise velocity for the three models.

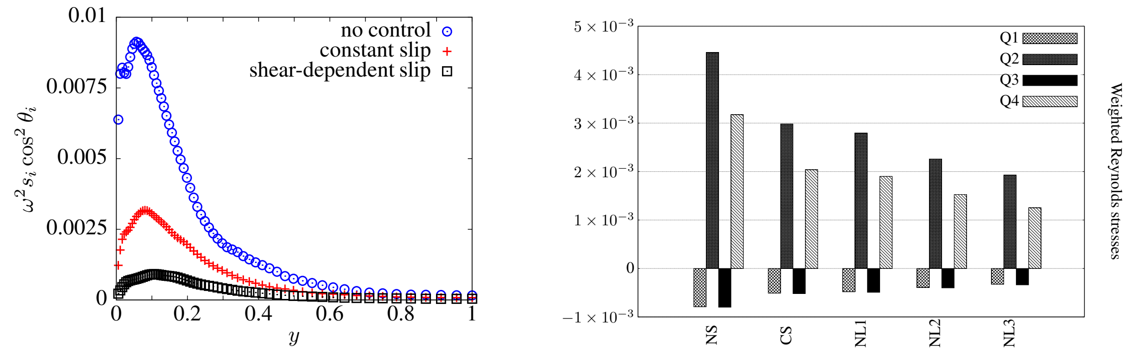

Figure 2. Left: Enstrophy production term evolution for the three configurations. Right: Weighted Reynolds shear-stress for no-slip case (NS), constant slip (CS) with $l_s = 0.02$ and shear-dependent slip (NL1) using $a_x = 0.001, b_x = 0.02$, (NL2) with $a_x = 0.005, b_x = 0.02$ and (NL3) with $a_x = 0.01, b_x = 0.02$

carried out following [5], leading to a transcendental equation solved for the drag-reduction rate $\mathcal{R} = 1 - (u_\tau/u_{\tau 0})^2$:

$$a_x(1-\mathcal{R})\frac{Re_{\tau 0}^3}{Re_p} + b_x Re_{\tau 0} = \left(\frac{1}{\kappa}\ln Re_{\tau 0} + F\right)\left(\frac{1-\sqrt{1-\mathcal{R}}}{1-\mathcal{R}}\right) - \frac{1}{\kappa}\frac{\ln\sqrt{1-\mathcal{R}}}{\sqrt{1-\mathcal{R}}}. \quad (2)$$

where Re_p is the Reynolds number based on the Poiseuille velocity flow and δ, while κ is the von Kármán constant. Eq.(2) is solved numerically and the results are consistent with the DNS implementation in the Incompact3d code [7]. Fig.1 shows that for a fixed b_x value, as $a_x \to 0$, the drag reduction rate values from the constant slip length model [8] are recovered. Therefore the shear-dependent slip length model can enhance the drag reduction rate. Also, the fluctuations are more significantly decreased as the parameter a_x is increased for a fixed b_x value, implying an increase in the effective slip length for the shear-dependent model. The PDF for the streamwise fluctuating velocity in the no-slip, constant and shear-dependent slip length models are computed. These are consistent with the r.m.s. of the vorticity fluctuations. The excursions of u become less pronounced for finite slip length models. The intense fluctuations for u in the high speed streaks give rise to strong spanwise vorticity fluctuations. This is justified by the fact that regardless of the control, ω_z is dominated by $-u'_y$ [3]. Thus, if such fluctuations are damped, drag should also be reduced. The control effect is also analysed through the orientation of the vorticity vector $\boldsymbol{\omega}$ and the principal strain rates denoted by s_i, $i \in [\![1, 3]\!]$. The associated eigenvectors are the principal axes denoted by $\boldsymbol{e}_i$. The angles between $\boldsymbol{\omega}$ and $\boldsymbol{e}_i$ are defined by $\cos\theta_i = \boldsymbol{\omega}\cdot\boldsymbol{e}_i/(\|\boldsymbol{\omega}\|\|\boldsymbol{e}_i\|)$ subsequently related to the enstrophy production term, $\omega_i S_{ij}\omega_j = \omega^2 s_i \cos^2\theta_i$ [6]. In a control framework, the evolution of enstrophy is often used to highlight its effect on the system. Near the wall, the finite slip length models act to reduce the enstrophy production term. The impact of control is further quantified by computing the weighted contributions of the Reynolds shear-stress arising in the FIK identity [4] for each quadrant. In Fig.2, Q2 and Q4 events are reduced when finite slip length models are used. The strength for this reduction being greater for the shear-dependent slip length compared to the constant slip model.

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