SPECTRA OF TURBULENT ENERGY TRANSPORT IN CHANNEL FLOWS

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Abstract To reveal the scale-dependences of the transport of turbulent energy in a channel flow, the constituents of the budget equation of turbulent energy for the Fourier modes of velocity fluctuations are computed by using direct numerical simulations. At each height in the buffer and overlap regions, the transport in the wall-normal direction by the turbulent convection provides energy to the fluctuations at small scales, but takes it away from those at large-scales. Furthermore, energy taken from the large-scales in the overlap region is carried upward to the center of channel and also downward to the vicinity of the wall. This downward transport is expected to cause the anomaly of the turbulent intensity and the constituents of the budget equation near the wall. The transport between scales and their scaling will also be discussed in the talk.

BUDGET EQUATION FOR FOURIER MODES

The transport of turbulent energy plays a central role for maintaining statistically steady turbulent flows. In wall-bounded flows, a net energy flux in the inhomogeneous wall-normal direction occurs, and it contributes to the budget of turbulent energy at each height. Especially in the logarithmic layer, the flux is almost constant, and the production by the mean shear and viscous dissipation balance in average although it is also known that the former slightly excesses the latter.[1] However, this does not imply that the energy is transported uniformly at all the scales, but is done depending on scales, as expected by considering the dynamics of turbulent structures.[2] In addition, the usual budget equation does not contain any contribution from the transport between scales. In wall-bounded flows, anisotropic large-scale fluctuations exchange energy in a more complicated manner than the small-scale isotropic ones. Recently, the energy fluxes in the combined physical/scale space is deduced from an extended Kolmogorov equation.[3] Being motivated by their study, the present one aims to extract the scale-dependences of the energy transport by employing a traditional Fourier spectral technique.

We consider the transport of turbulent energy in a fully developed turbulent channel flow. We use x, y, and z for the streamwise, wall-normal, and spanwise coordinates. Let $U(y)$ denote the mean streamwise velocity, and u, v, and w the velocity fluctuations in the x, y, and z directions, respectively, and p the pressure fluctuation. The channel half width is h. We may use u_1, u_2 and u_3 interchangeably with u, v and w. The Fourier transform of $u_i(x, y, z)$ in the wall-parallel directions is expressed by $\widehat{u}_i(\boldsymbol{k}, y)$, where $\boldsymbol{k} = (k_x, k_z)$ is the wavenumber-vector whose components are the wavenumbers in the x and z directions. One may then derive the budget equation for the Fourier modes of the velocity fluctuations, as follows,

$$\frac{1}{2}\frac{\partial\overline{|\widehat{u}_i|^2}}{\partial t} = P\delta_{1i} + T_i^s + T_i^p + \Pi_i + \varepsilon_i + D_i, \tag{1}$$

where

$$P(\boldsymbol{k}, y) = \Re\left\{-\overline{\widehat{u}\widehat{v}^*}\frac{\mathrm{d}U}{\mathrm{d}y}\right\}, \quad T_i^s(\boldsymbol{k}, y) = \Re\left\{\sum_{j=1}^{3}\overline{\partial_j\widehat{u}_i(\widehat{u_iu_j})^*} - \frac{1}{2}\frac{\mathrm{d}\overline{\widehat{u}_i(\widehat{u_iv})^*}}{\mathrm{d}y}\right\}, \quad T_i^p(\boldsymbol{k}, y) = \Re\left\{-\frac{1}{2}\frac{\mathrm{d}\overline{\widehat{u}_i(\widehat{u_iv})^*}}{\mathrm{d}y}\right\},$$

$$\Pi_i(\boldsymbol{k}, y) = \Re\left\{-\frac{1}{\rho}\overline{\widehat{u}_i(\partial_i\widehat{p})^*}\right\}, \quad \varepsilon_i(\boldsymbol{k}, y) = -\nu\left((k_x^2 + k_z^2)\overline{|\widehat{u}_i|^2} + \overline{\left|\frac{\partial\widehat{u}_i}{\partial y}\right|^2}\right), \quad D_i(\boldsymbol{k}, y) = \frac{1}{2}\nu\frac{\mathrm{d}^2\overline{|\widehat{u}_i|^2}}{\mathrm{d}y^2},$$

and $\partial_1 = ik_x$, $\partial_2 = \partial/\partial y$ and $\partial_3 = ik_z$. The over-line means a temporal average, and the superscript '*' stands for the complex conjugate. If the flow is statistically steady, the terms in the r.h.s. of (1) balance. Here P is the spectrum of the production by the mean shear, T_i^s is the spectrum of the transport between scales, T_i^p is the spectrum of the transport in the wall-normal direction by turbulent convection, Π_i is the spectrum of the pressure term, and ε_i and D_i are the viscous dissipation and diffusion, respectively, where ν is the kinematic viscosity. Since $\sum_{\boldsymbol{k}} T_i^s(\boldsymbol{k}, y) = 0$ for any y, this term disappears in the usual budget equation which is obtained by summing the terms in eq. (1) over all the wavenumbers. An advantage of eq. (1) is that the transport terms are given in the form of cospectra which represent their scale-dependences.

DIRECT NUMERICAL SIMULATIONS

Direct numerical simulations (DNSes) have enabled us to study the budget of turbulent energy quantitatively in detail.[4, 1] DNSes of turbulent channel flows for several Reynolds numbers have been carried out to compute the constituents of the spectral budget equation (1). The size of the numerical domain is $L_x \times L_y \times L_z = 32\pi h \times 2h \times 2\pi h$ in common to all the simulations. For spatial discretization, Fourier spectral method is employed for the wall-parallel directions, and

	$N_x \times N_y \times N_z$	$\Delta x^+ (= \Delta z^+)$, $(\Delta y/\eta)_{\max}$	$T_s u_\tau / h$	h^+
C200	$4096 \times 129 \times 256$	5.07, 1.23	210	207
C500	$8192 \times 257 \times 512$	6.08, 1.17	34	496
C900	$16384 \times 513 \times 1024$	5.60, 0.914	2.0 $^{*)}$	912

Table 1. Parameters of the simulations. The superscript '+' means the normalization with a wall-unit. Here u_τ is the friction velocity, η the Kolmogorov's length-scale. For the spatial resolution, grid spacings Δx, Δy and Δz in the three directions are shown in a wall- or viscous unit. The time interval used in analysis for each case is shown as T_s. $^{*)}$An enough number of samples will be obtained by the meeting.

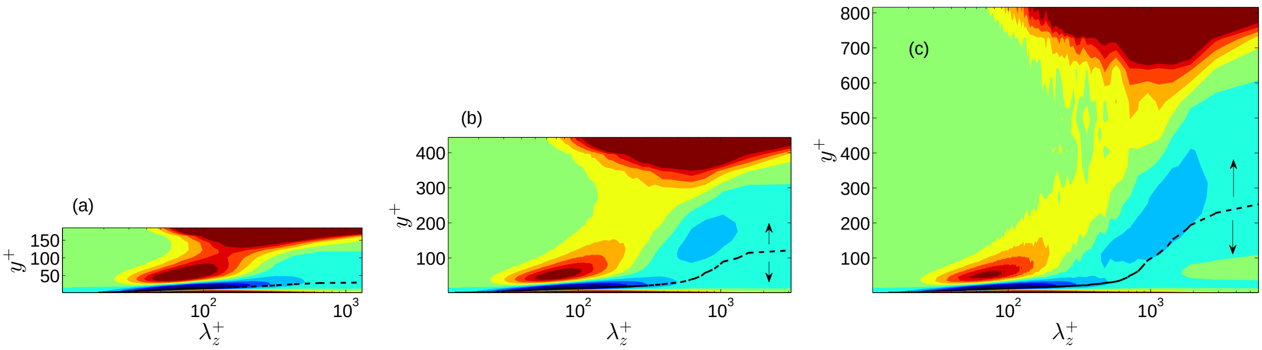

Figure 1. Premultiplied one-dimensional spectra $k_z T^p_{1,1d}$ as functions of λ_z^+ and y^+ for (a) C200, (b) C500 and (c) C900. They are normalized by the production at each height, and $T^p_{1,1d} = 0$ on the border between green (positive) and blue (negative) areas. Dashed lines are the contours of $\Theta^p_{1,1d} = 0$, and the arrows indicate the direction of the flux.

an 8th-order compact finite difference scheme on a nonuniform grid for the wall-normal one. A semi-implicit 3-step 3rd-order Runge-Kutta scheme was used for time marching. The numerical conditions are summarized in Table 1.

SPECTRA OF ENERGY TRANSPORT IN THE WALL-NORMAL DIRECTION

The scale-dependence of the transport of turbulent energy in the wall-normal direction is presented here. Figure 1 shows the one-dimensional spectra of $T_1^p(\boldsymbol{k}, y)$, $T^p_{1,1d}(k_z, y) = \sum_{k_x} T_1^p(\boldsymbol{k}, y)$, as functions of the wavelength $\lambda_z = 2\pi/k_z$ and y. The zero level of the energy flux, $\Theta^p_{1,1d}(k_z, y) = -\int_0^y T^p_{1,1d}(k_z, y')\mathrm{d}y' = \Re\left\{\frac{1}{2}\sum_{k_x} \overline{\widehat{u}(\widehat{uv})^*}\right\}$, is also shown together as dashed lines. The transport T_1^p is positive at smaller, but is negative at larger spanwise scales in the buffer and overlap layers. This means that fluctuations at small-scales receive energy from the lower ones while those at large-scales give it to the upper ones, where the flux directs upward. It is found that the flux directs towards the center of channel above the buffer layer at the most of scales, but may be opposite at very large scales. As Reynolds number is increased, the range where the flux directs downward extends more into the overlap layer. This downward transport which carries energy from the overlap layer to the buffer and viscous sublayer seems to cause the well-known anomaly of the wall-scaling of the Reynolds stress and the constituents of the budget equation near the wall.[6, 5, 1] In addition, figures 1(b,c) indicate the self-similarity of the eddies contributing to the spatial energy transport in the overlap layer, roughly in the range of $150 < y^+$ and $y/h < 0.5$.

As shown above, the spectral budget equation gives rich information on the transport of turbulent energy. We will discuss the scale-dependence of T_i^s as well as T_i^p, and their scaling in the overlap region in the talk.

References

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