UNSTABLE PERIODIC MOTION IN LARGE EDDY SIMULATION OF HOMOGENEOUS, ISOTROPIC TURBULENCE

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Abstract We study unstable, time-periodic solutions in Large Eddy Simulation (LES) of Homogeneous, Isotropic Turbulence (HIT). The turbulence is forced at large spatial scales by an external body force which induces four vortex columns in a rectangular domain with periodic boundary conditions. The dissipation range dynamics is represented by the Smagorinsky model. Both the kinematic viscosity and the Smagorinsky constant are used as a homotopy parameters connecting steady, laminar flow to turbulent inertial range dynamics. We find several families of periodic solutions, some bifurcating from the steady flow and some filtered from more turbulent dynamics, and attempt to track these solutions to the limit of fully developed turbulence.

INTRODUCTION

According to the paradigm of dynamical systems, we can regard turbulent flow as a manifestation of chaotic motion in a high-dimensional phase space. The hypothetical "turbulent attractor" then contains infinitely many Unstable Periodic Orbits (UPOs), that may be thought of as the building blocks of turbulence [1]. In this sense, computing periodic orbits serves a purpose similar to that of computing Empirical Orthogonal Functions, namely to decompose the complicated, spatio-temporal patterns of turbulent fluid motion into relatively simple components. An advantage of periodic orbits is that they capture the undiluted dynamics of the Navier-Stokes equation. They are reproducible and can be computed as a function of system parameters such as the Reynolds number. While they convey a lot of information about the turbulent flow, unstable periodic orbits are hard to compute. Even when using the most efficient algorithms from computational science, and focusing on fairly low Reynolds numbers and small computational domains, the extraction of even a single periodic orbit requires many CPU hours and some serendipity. For this reason, the study of fully developed turbulence, with inertial range dynamics insensitive to the forcing at large scales, has so far been out of reach. In the current work, we propose to use LES to address this problem. Using this classical technique from numerical fluid simulation, we can drastically reduce the number of degrees of freedom, and thereby the size of the nonlinear systems of equations we need to solve when searching for periodic solutions. The ultimate goal of this project is to study the dynamical processes that contribute to energy transfer in the inertial range, using unstable periodic orbits as templates.

LES WITH THE SMAGORINSKY MODEL

The starting point of LES is the application of a spatial filter to the velocity field. All motion on scales smaller than a given cut-off, refered to as the grid scale, is modeled by an effective "eddy viscosity", leading to the dynamical equations

$$\bar{u}_t = -\bar{u}\nabla\bar{u} - \nabla\left(\frac{\bar{p}}{\rho} + \frac{\tau}{3}\right) + 2\nabla\cdot\left([\nu_T + \nu]\,\bar{S}\right) + f \qquad \nabla\cdot\bar{u} = 0$$

where the over bars denote the application of the spatial filter, ν_T is the eddy viscosity, $\underline{S}$ is the rate-of-strain tensor and $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j$. In the Smagorinsky model, ν_T is expressed as $\nu_T = (C_s\delta)^2\sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$ (summation over i and j implied), with $C_s = 0.18$ a constant and δ the spatial cut-off length scale.

We consider a constant external force f that takes the form of four counter rotating vortices, aligned with the vertical axis, as described by Yasuda *et al.* [2]. It is shown in Fig. 1(left). Yasuda *et al.* studied the evolution of the velocity field for zero molecular viscosity, i.e. $\nu = 0$, and found that it is quasi-cyclic, as illustrated in Fig. 2(left). In one phase, the large-scale vortical structures gather strength while on smaller scales the flow is quiescent. Subsequently, a cascade process can be observed, in which smaller scale vortical structures grow in the strain fields induced by larger scale structures. This corresponds to a transfer of energy towards the grid scale, where it is dissipated by the eddy viscosity. Fig. 1 also shows the equilibrium flow induced by the forcing at large molecular viscosity and a snap shot of the quasi-cyclic behaviour in developed turbulence. Using spatial band-pass filtering, vortical structures can be distinguished on three different spatial scales.

UPO COMPUTATION

The prevalence of approximately cyclic behaviour in LES simulation suggests that we may be able compute UPOs representing the turbulent energy cascade. Due to the complexity of the flow at zero molecular viscosity and high spatial

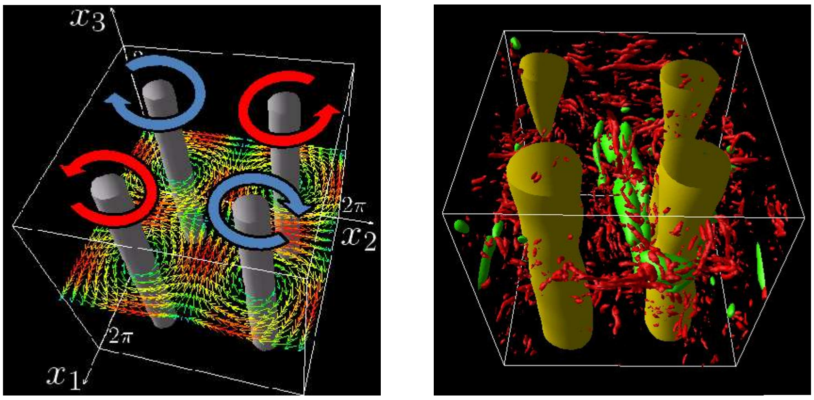

Figure 1. Left: visualisation of the external force and the viscous equilibrium flow at low Reynolds numbers. Four counter rotating large-scale vortices are induced by the steady forcing. The equilibrium velocity field is shown in the mid plane with colour indicating magnitude from small (green) to large (red). Right: snap shot of quasi-cyclic motion at zero molecular viscosity, taken at a local maximum of the energy input rate, time t_1 in Fig. 2 below. Shown are isosurfaces of vorticity after application of band-pass filters for highlighting large-scale structures (yellow), intermediary-scale structures (green) and small-scale structures (red).

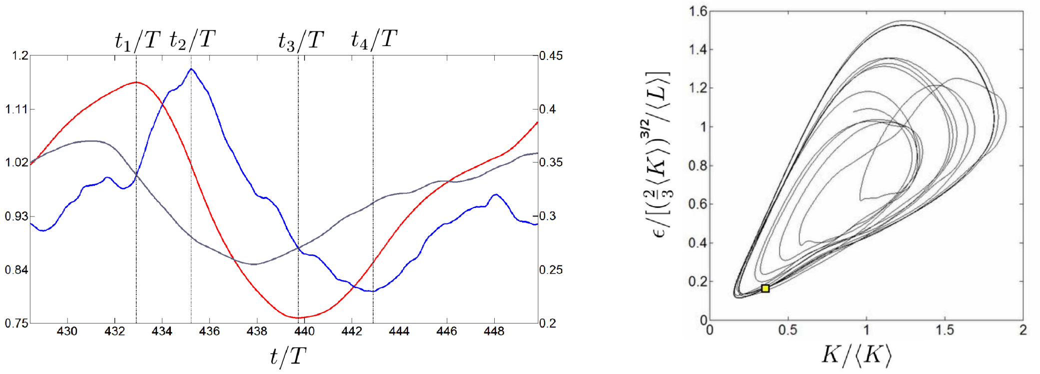

Figure 2. Left: quasi-cyclic behaviour for zero molecular viscosity. Shown are the normalized kinetic energy $K/<K>$ in black, the normalized energy input rate $I <L> /(\frac{2}{3} <K>)^{3/2}$ in red and the normalized energy transfer to sub grid scales $\epsilon <L> /(\frac{2}{3} <K>)^{3/2}$ in blue. The brackets denote the time average, $<L>$ is the integral length and T the large eddy turnover time. Right: example of a UPO computed for nonzero ν, along with a turbulent orbit.

resolution, a direct computation of such solutions is impossible. Instead, we compute UPOs at intermediate values of ν, or at $\nu = 0$ and artificially high values of C_s, and use arclength continuation to approach the regime of developed turbulence. The algorithmic and computational aspects of this homotopy approach are explained in Ref. [1]. It has proven successful before in the study of box turbulence by van Veen *et al.* [3]. However, in that study spatial symmetries were imposed to reduce the number of degrees of freedom. In our current study, no such restrictions are applied. Consequently, even for moderate resolution there are millions of unknowns to solve for when tracking the UPOs. We are currently tracking several UPOs in both homotopy parameters. An example is shown in Fig. 2(right), which shows a projection of a UPO computed at coarse spatial resolution and $\nu \approx 0.05$ onto the energy input rate and the rate of energy transfer to sub grid scales. In the course of the next few months we expect to approach the limit of fully developed turbulence, and use the UPOs to study the vortical dynamics of energy transfer.

References

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