AN EXAMINATION OF KOLMOGOROV'S REFINED SIMILARITY HYPOTHESES FOR ACTIVE SCALAR IN COMPRESSIBLE TURBULENCE

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Abstract Using direct numerical simulations of isotropic compressible turbulence driven by large-scale solenoidal force (up to 1024^3 grid points), we examine the Kolmogorov's refined similarity hypotheses (RSH) as applied to active scalar, i.e. temperature, with Prandtl number of order one. The three-dimensional compressible Navier-Stokes equations are solved by adopting a hybrid method for space and a second-order Runge-Kutta technique for time. The stationary turbulent Mach number, M_t, and Taylor microscale Reynolds number, Re_λ, vary from 0.3 to 1.0 and 123 to 255, respectively. The two-dimensional contours of temperature dissipation field show that at low M_t the field is dominated by vortices structures while at high M_t it is full of small-scale shocklets structures. When Re_λ increases, the random distribution of shocklets is reinforced, and thus, the field tends to local isotropy at small scales. According to the scaling exponents obtained in our simulations, the probability distribution of the normalized temperature increment is basically the same for the temperature fields, however, they are not close to Gaussian. Furthermore, the usage of the scaling exponents from standard RSH theory [1,2] shows that the new probability distribution behaves rather different, suggesting the failure of RSH for temperature.

INTRODUCTION

Understanding the universal features of turbulence is a formidable problem in mathematics and physics. The original K41 theory relied on the global average of energy dissipation rate to predict the scaling properties of energy spectrum and others. Based on a set of refined similarity hypotheses (RSH), it was later extended to the K62 theory by considering strong intermittency in local energy dissipation rate. Direct examinations of these hypotheses have been carried out experimentally [3] and numerically [4]. Analyses of the RSH for passive scalar advected by incompressible turbulent flows have been performed and examined by experimental and numerical data [1,2] as well. In compressible turbulence, the temperature field has complicated nonlinear couplings with velocity field, and thus, is called as active scalar, which belongs to fully nonlinear problems. In this study, we carry out numerical investigations on the following question: whether the temperature in compressible turbulence, as an active scalar, obeys the RSH theory? More details on this study can be found in Ref. 5.

NUMERICAL RESULTS

Figure 1 presents three two-dimensional contours of temperature dissipation field from the simulated flows of (M_t, Re_λ)=(0.3, 125), (1.0, 123) and (1.0, 255). It shows that in (a) the field is occupied by the randomly distributed vortices, and there appear few discontinuities characterizing by shocklets. In (b) the obvious shocklets are in coexistence with vortices, while in (c) the field is dominated by the small-scale shocklets, with random distribution. We find that as Re_λ increases, the field tends to local isotropy at small scales. In fact, the ensemble averages of skewness for the temperature dissipation rate are $\langle S_\chi \rangle = -0.46$, -1.22 and -0.71 for (a), (b) and (c), respectively. Here $\chi \equiv \kappa(\nabla Te)^2$ is the temperature dissipation rate.

In our simulations, the scaling exponents computed from the conditionally averaged temperature increment, $\langle \delta_r Te|_{\chi_r,\epsilon_r}\rangle$, over the locally averaged kinetic energy dissipation rate ϵ_r and temperature dissipation rate χ_r, and the separation distance between two points r are Z_m, 0.5 and 1.0, respectively. Here the expression of Z_m is obtained by data fitting.

$$Z_m(M_t) = -\frac{M_t}{a} + \frac{1}{b}, \qquad (1)$$

where $a = 5$ and $b = 10$. The normalized temperature increment is then defined as $\beta_s \equiv (\delta_r Te|\chi_r, \epsilon_r)/(\chi_r^{1/2}\epsilon_r^{Z_m} r)$. In the left panel of Figure 2 we plot the probability distribution function (PDF) of β_s in the inertial range, where σ_s is the standard derivation. It is found that the probability distribution is almost the same for the three simulated temperature fields, implying some university of distribution. The discrepancy at large β_s is possible caused by statistical variability. However, the probability distribution significantly deviates from Gaussian even that β_s is at small amplitudes. The right panel of Figure 2 shows the PDF of another normalized temperature increment $\varphi_s \equiv (\delta_r Te|\chi_r, \epsilon_r)/(\chi_r^{1/2}\epsilon_r^{-1/6} r^{1/3})$, which is defined according to the values of scaling exponents in Ref. 2. Here ϖ_s is the standard derivation of φ_s. We observe that for each simulated temperature field, the probability distribution of φ_s is quite different. This reveals that the classical RSH theory for passive scalar in incompressible turbulence fails to explain the active scalar like temperature in compressible turbulence.

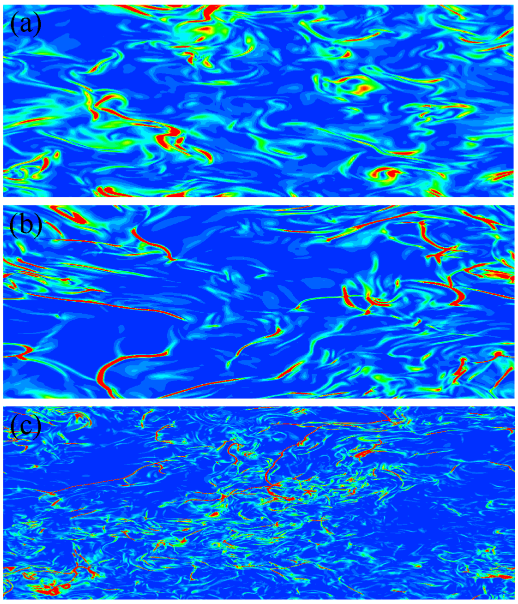

Figure 1. Two-dimensional contours of temperature dissipation field. (a) $M_t = 0.3$ and $Re_\lambda = 125$; (b) $M_t = 1.0$ and $Re_\lambda = 123$; (c) $M_t = 1.0$ and $Re_\lambda = 255$.

In summary, through simulations of isotropic compressible turbulence, we find that the Kolmogorov's refined similarity hypotheses are not applicable to temperature, which is an active scalar in compressible turbulence. The reason is that in compressible turbulent flows the cascade of temperature is governed by the compressive component of velocity. In other words, because of the motions of rarefaction and compression caused by shocklets, the temperature increment $\delta_r Te$ at small scales is proportional to r by the Taylor expansion, rather than $r^{1/3}$ by the Kolmogorov-Obukhov-Corrsin theory.

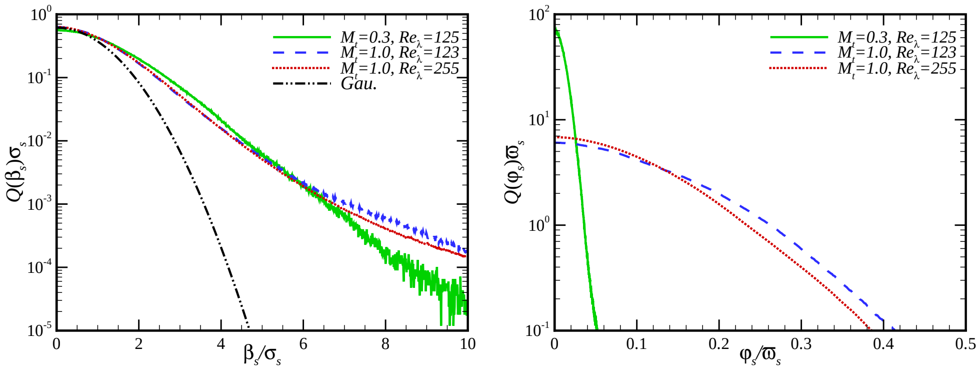

Figure 2. Probability distribution of normalized temperature increment by its standard deviation in the inertial range. (a) $\beta_s = (\delta_r Te|_{\chi_r,\epsilon_r})/(\chi_r^{1/2}\epsilon_r^{Z_m} r)$, where $Z_m = 0.04$ for $M_t = 0.3$, and $Z_m = -0.1$ for $M_t = 1.0$; (b) $\varphi_s = (\delta_r Te|_{\chi_r,\epsilon_r})/(\chi_r^{1/2}\epsilon_r^{-1/6} r^{1/3})$.

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