TURBULENT MHD CHANNEL FLOWS UNDER STREAMWISE MAGNETIC FIELD

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Abstract A streamwise magnetic field leads to turbulent drag reduction in channel flow of a conducting liquid due to the selective Joule damping of certain flow structures. Near the walls, the turbulent mean velocity profile retains the logarithmic layer but the von Karman constant decreases with increasing magnetic field strength. In the outer region, the flow is characterized by persistent streaky structures of large streamwise extent, which lead to a rather flat mean velocity profile. In addition, the streamwise velocity fluctuations develop a pronounced second peak upon increasing the magnetic induction as well as a second logarithmic layer that increases in steepness.

INTRODUCTION

Flows of electrically conducing fluid under magnetic field (magnetohydrodynamic or MHD flows) are encounter in a variety of industrial and engineering applications such as continuous steel casting, crystal growth and flows in cooling blankets in fusion reactors. The magnetic field is either present in these systems as a natural attribute of the physics (fusion reactors) or is introduced as a tool to provide flow control (steel casting). The fluid motion generates electric currents, which, due to the interaction with the applied magnetic field, induce Lorentz force acting on the flow. As a result of this interaction the flow properties are modified. The interaction between the fluid and magnetic field is twofold. On one hand, the magnetic field can strongly modify the basic velocity profile, so that thin boundary layers – the so called Hartmann layers at walls normal to the field and Shercliff layers at walls parallel to the field – are formed. On the other hand, there is a selective damping of turbulent fluctuations by the magnetic field. The preferential direction is set by the magnetic field vector, so that turbulent eddies aligned parallel to the field are not suppressed, whereas eddies perpendicular to the field vector experience the strongest suppression. If the field becomes sufficiently strong, the flow can form a structure with anisotropic, even quasi-$2D$, alignment of turbulent vortices. Ultimately, if the quasi-$2D$ state cannot sustain the mechanism of turbulence regeneration, the flow may become fully laminar, even at high Reynolds numbers. In our prior studies we have performed a series of DNS and LES to investigate periodic channel flow under homogeneous magnetic field applied in the spanwise direction. In that case the magnetic field does not modify the laminar basic velocity (a counterpart to the classical Hartmann flow with wall-normal field) but suppresses turbulent fluctuations. It has been shown that the turbulent velocity profile is modified because of this damping. Namely, it becomes steeper as the magnetic field increases and deviates from the classical log-layer behavior. We have also shown that the velocity profile reveals a strong contribution from the linear dependence on the wall-normal coordinate y. Using the results of DNS and LES as a database we have constructed an extension to the mixing-length model [1]. The model has shown a very close match to the results of numerical simulations in a broad range of Reynodls and Hartmann numbers as well as for duct flow at very high parameters [4].

MATHEMATICAL MODEL, PROCEDURE AND PARAMETERS

In the present work we consider the case of channel flow under streamwise magnetic field. This configuration has been studied by the authors in [3], where the suppression of turbulent fluctuations and the growth of secondary perturbations were analyzed in a series of DNS. Here we perform additional DNS and LES in a broader range of Reynolds and Hartmann numbers and study the properties of the turbulence at increasing magnetic field strength.

We consider flow of incompressible, Newtonian, electrically conducting fluid (e.g. liquid metals) in a periodic channel between two perfectly insulating walls. Using the assumption of small magnetic Reynolds number Re_m, the flow is described by the quasi-static approximation of MHD equations [2]. The non-dimensional parameters are the Reynolds $Re \equiv Ua/\nu$ and Hartmann $Ha \equiv Ba\,(\sigma/\rho\nu)^{1/2}$ numbers. Here U is the mean flux velocity, a is the channel half-width, σ is the electrical conductivity and B is the applied magnetic field. The governing equations are the Navier-Stokes equations with an additional Lorentz-force term $\boldsymbol{j} \times B$, where $\boldsymbol{j}$ is the induced electric current. The equations are solved numerically with our spectral DNS/LES solver [1], the dynamic Smagorinsky model is used for the sub-grid closure.

For the modeling of the velocity profile in the case of a spanwise field we have used a modification of the classical mixing-length model for the turbulent stress $-\langle u'v' \rangle = l_m^2(y)\left(\frac{du}{dy}\right)^2$ proposed by Prandtl[5] , which relates the fluctuating velocity components u' and v' to the mean velocity distribution. For the spanwise field it turned out that the mixing length l_m is constrained by another length scale L constructed from the Joule damping time and friction velocity, i.e., the mixing length is the harmonic mean $l_m = \frac{\kappa y L}{y+L}$ of the wall distance and L. We explore if this approach can also be used to represent the velocity distribution when the field is in the streamwise direction.

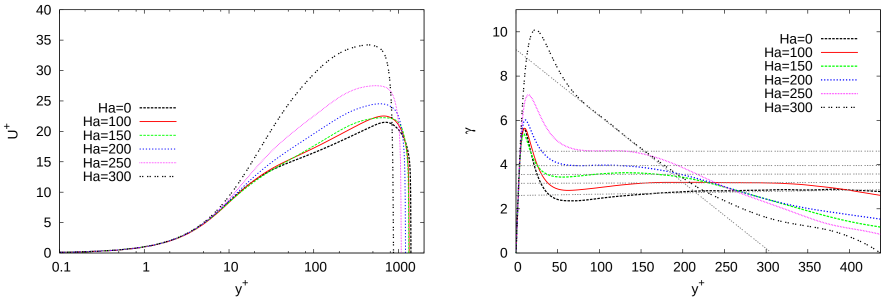

Figure 1. Effect of the streamwise magnetic field on the mean velocity. Results of simulations at $Re = 13334$ are shown in wall-units for time- and domain-averaged profile of the mean velocity (left) and compensated profile $\gamma = y^+ dU^+/dy^+$ (right). Dotted lines are fitted by "eye-balling" to help identifying the von Karman constant κ.

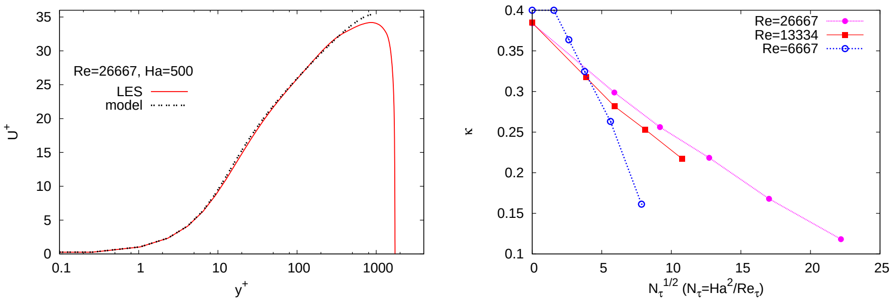

Figure 2. Mean velocity profile, results of LES at $Re = 26667$ and $Ha = 500$ vs. mixing-length model (left), von Karman constant κ vs. the shear-based Stuart number $N_\tau = Ha^2/Re_\tau$ for $Re = 6667$, 13334 and 26667 (right).

RESULTS AND DISCUSSION

Simulations have been performed for three values of Re, namely 6666, 13334 and 26667. In each case the Hartmann number Ha is varied in the limits between 0 and a certain maximum value, at which complete flow laminarization is observed. The domain size is $4\pi \times 2\pi \times 2$ in the streawise, spanwise and wall-normal directions, correspondingly, the numerical resolution is 256^3 points. The simulations were running for about 500 convective units to gather well converged statistics in terms of mean velocity profile and fluctuating quantities, such as components of the Reynolds stresses.

It has been found that there is a clearly pronounced modification of the basic velocity profile by the streamwise field B, as shown in fig. 1 (left) for the case $Re = 13334$. Further analysis also shows that there is a gradual change in the von Karman constant κ, which is reflected in fig. 1 (right) showing compensated velocity profile $\gamma = y^+ dU^+/dy^+$ as a function of Ha. In contrast to the spanwise magnetic field, there is no linear increase of γ with y^+. Close to relaminarization, the plateau in γ is lost, and one can identify a linear range with decreasing γ.

We have also attempted to fit the DNS/LES velocity profiles to a mixing length model with κ and L as free parameters. Fig.2(left) shows the case $Re = 26667$ and $Ha = 500$. The parameter L does not help to capture the profile and has to be set to infinity. The good agreement over a wide y^+-range is achieved by tuning κ. We have carried out such fits to the mixing length model for all available data sets, and consistently found that there is no cut-off by L. One may speculate about the universality of the observed dependence of κ on the flow parameters, which should effectively include inertia and Lorentz forces only. This would suggest a universal dependence of κ on the interaction or Stuart number based on the friction velocity. Fig.2(right) indicates that such a universal dependence may indeed exist provided that the Reynolds number is sufficiently high. The observed flattening of the velocity near the middle requires a different modeling approach.

References

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