LAGRANGIAN RAYLAIGH-BÉNARD CONVECTION

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Abstract Using passive tracers as sensors, we obtain Lagrangian measurements of tracers position, velocity and temperature in Rayleigh-Bénard convection at $Ra \sim 10^7 \div 10^9$. We report on statistics of temperature, velocity, and heat transport (Nusselt number). We observe that the Nusselt number is characterized by a largely intermittent behavior, likely due to the interaction of temperature with turbulent velocity fluctuations.

INTRODUCTION

Understanding fluid motion and heat transport in turbulent thermal convection is crucial for progress in industrial, geophysical and environmental applications. A large proportion of previous studies on turbulent thermal convection (Rayleigh-Bénard) focuses on the estimate of local and global heat transfer properties performing Eulerian (experimental or numerical) measurements. It has been recently proven that Lagrangian measurements of turbulence can provide new and more useful insights into the local flow structure, the pair particle dispersion or the local particle acceleration. Yet, Lagrangian measurements are particularly suited for studying flows where coherent structures and mixing are important, e.g. plumes mixing temperature in thermal convection. However, Lagrangian measurements in numerical simulations of thermal convection are extremely rare, with the only exception given by [2]. Differently from the canonical Rayleigh-Bénard, [2] assumed free-slip top/bottom wall and focused mainly on pair and multi-particl dispersion by velocity fluctuations. So far, Lagrangian thermal properties of the flow were slightly accounted. Only recently, Gasteuil et al. [1] developed a smart sensor able to perform Lagrangian measurements of position, velocity and temperature in thermal convection. Our aim here is to perform Direct Numerical Simulations (DNS) of Rayleigh-Benard convection and Lagrangian tracking of passive particles in a flow configuration close to that of the experiments [1]. Our numerical simulations provide combined data of velocity, temperature, and heat transport dynamics which can be used as reference/benchmark data to tune and explain experimental results.

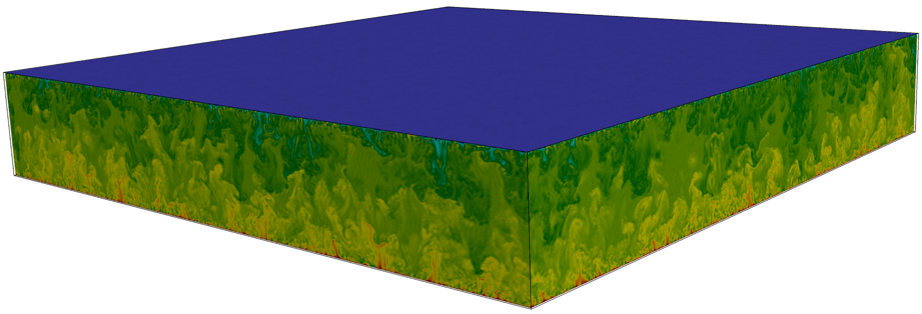

Figure 1. Temperature distribution for $Ra = 10^9$. Red indicates regions of high temperature, whereas blue indicates regions of low temperature.

GOVERNING EQUATIONS AND NUMERICAL MODELING

We consider an incompressible and Newtonian turbulent flow of water confined between two rigid boundaries. The bottom wall is kept at uniform high temperature (θ_h), whereas the top wall is kept at uniform low temperature (θ_c). The size of the computational domain is $L_x \times L_y \times L_z = 2\pi h \times 2\pi h \times 2h$ (in x, y and z, respectively), where h is the half-channel height. A sketch of the computational domain, along with the distribution of temperature for $Ra = 10^9$ is shown in Fig. 1. Mass, momentum and energy equations in dimensionless form and under the Boussinesq approximation are:

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + 4\sqrt{\frac{Pr}{Ra}}\nabla^2 \mathbf{u} - \delta_{i,3}\theta, \tag{2}$$

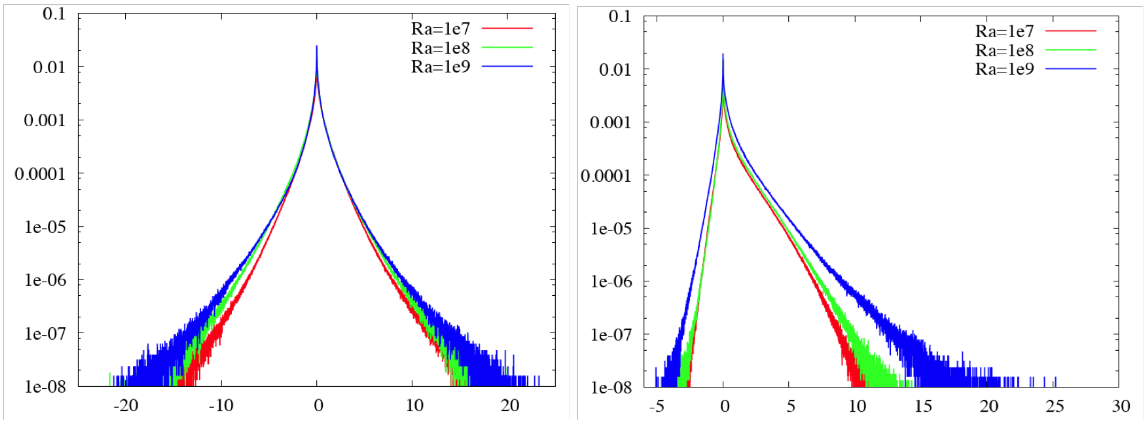

Figure 2. Probability density function (pdf) of the horizontal $Nu^L x$ (left) and vertical $Nu^L z$ (right) Nusselt numbers for turbulent thermal convection at different Rayleigh number.

$$\frac{\partial \theta}{\partial t} + (\mathbf{u} \cdot \nabla)\theta = -\nabla p + \frac{4}{\sqrt{PrRa}} \nabla^2 \theta, \tag{3}$$

where u_i is the i^{th} component of the velocity vector, θ is temperature, p is pressure, $\delta_{1,3}\theta$ is the buoyancy force (acting in the vertical direction only) that drives the flow. Eqs. 1-3 have been obtained using $L_{ref} = h$ as reference length, $u_{ref} = \sqrt{g\beta\Delta\theta/2h}$ as reference velocity, $\theta_{ref} = \Delta\theta/2$ as reference temperature and $p = \rho g\beta\Delta\theta/2h$ as reference pressure. Density ρ, kinematic viscosity ν, thermal diffusivity k and thermal expansion coefficient β are evaluated at the mean fluid temperature $\theta_m = 29^oC$. Two dimensionless numbers appear in Eqs. 1-3: the Prandtl number $Pr = \nu/k$ and the Rayleigh number $Ra = (g\beta\Delta\theta(2h)^3)/(\nu k)$. In the present study we keep the Prandtl number $Pr = 4$ and we vary the Rayleigh number between $Ra = 10^7$ and $Ra = 10^9$. Periodic boundary conditions are imposed on velocity and temperature along the streamwise x and spanwise y directions; at the wall, (along the wall normal direction z) no slip conditions are enforced for the momentum equations while constant temperature conditions are adopted for the energy equation. The resulting set of equations are discretized using a pseudo-spectral method based on transforming the field variables into wavenumber space, through Fourier representations for the periodic (homogeneous) directions x and y, and Chebychev representation for the wall-normal (non- homogeneous) direction z[3]. We used up to $512 \times 512 \times 513$ grid points to discretize the computational domain. The dynamics of Lagrangian tracers is computed as $\dot{x} = u(\mathbf{x}, \mathbf{t})$. We injected $N_p = 1.28 \times 10^5$ Lagrangian tracers. Lagrangian particles are advanced in time using a 4th order Runge-Kutta scheme. Velocity and temperature at particle position are obtained by 6th order Lagrange polynomials.

RESULTS

One of the main results of our simulation is the computation of the local heat flux along the Lagrangian trajectories. The local heat flux is here quantified by the Lagrangian Nusselt number Nu^L:

$$Nu^L_{x,z} = 1 + \frac{h}{k\Delta\theta} u_{x,z}\theta' \tag{4}$$

where Nu^L_x and Nu^L_z are the horizontal and the vertical Nusselt numbers, respectively. In figure 2 we explicitly show the behavior of the probability density function (pdf) of Nu^L_x and Nu^L_z. These results are in nice agreement with experiments obtained by sampling the flow field with a millimetric sensor [1]. Within this framework, our simulations can be regarded as *gedanken* experiments performed using an infinitesimal sensor; hence, our data can be used to quantify size effects of sensors in real experiments. Further results will be also provided and discussed to characterize heat transport intermittency behavior in turbulent thermal convection.

References

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