

## CONVECTIVE RIPENING AND RAINFALL

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*Abstract* This paper discusses the evolution of the droplet size distribution for a liquid-in-gas aerosol contained in a Rayleigh-Bénard cell. It introduces a non-collisional model for broadening the droplet size distribution, termed ‘convective ripening’. The paper also considers the initiation of rainfall from ice-free cumulus clouds. It is argued that while collisional mechanisms cannot explain the production of rain from clouds with water droplet diameters of 20  $\mu\text{m}$ , the non-collisional convective ripening mechanism gives a much faster route to increasing the size of the small fraction of droplets that grow into raindrops.

### INTRODUCTION

The dynamics of the onset of rainfall from ice-free (‘warm’) cumulus clouds is poorly understood [1, 2, 3]. Coalescence of droplets which collide due to differential rates of gravitational settling is effective for droplets with radius  $a$  above  $50\mu\text{m}$ , and leads to a runaway growth to produce millimetre-scale raindrops [4]. Many clouds are found to contain droplets with radius approximately  $8 - 15\mu\text{m}$  [1, 2, 3], which result from primary condensation onto aerosol nuclei. For droplets in this size range, growth by collisional coalescence is slow because the product of the collision rate and the coalescence efficiency is low [1]. This makes it difficult to explain observations of the rapid onset of rainfall from warm cumulus clouds.

It is, therefore, desirable to formulate models for non-collisional growth of water droplets, in which some droplets are able to grow at the expense of others shrinking, by transferring water molecules between droplets as water vapour. It has been suggested that condensation processes may be able to cause the droplet size distribution to broaden due to fluctuations in the degree of supersaturation. This possibility has been addressed by numerous authors: see, for example, [5, 6, 7, 8, 9]. These investigations have used numerical simulations, and it is difficult to draw conclusions which are applicable to real clouds because of the limited range of size scales which can be simulated reliably. A difficulty with most of these models is that the droplets both grow and shrink as the supersaturation fluctuates. This work, however, introduces a model for which there is an asymmetry between growth and shrinkage.

This work considers how the process works in the simplest relevant model, which is an aerosol in a Rayleigh-Bénard convection cell. As well as having fewer physical parameters than a cloud, this system can be subject to a carefully controlled laboratory investigation.

### RIPENING IN A TURBULENT CONVECTION CELL

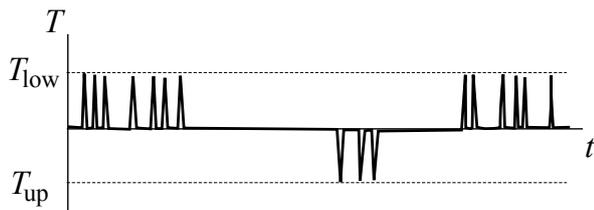
Now consider the response of an aerosol to convective motion in a Rayleigh-Bénard cell. This is a consequence of how the temperature changes along the trajectories of the aerosol droplets (which are assumed to be advected by the flow).

Turbulent convection in a Rayleigh-Bénard cell is reviewed in [10]. The upper and lower plates are at temperatures  $T_{\text{up}}$  and  $T_{\text{low}}$  respectively. The expectation value of the temperature is close to  $T_{\text{av}} = (T_{\text{up}} + T_{\text{low}})/2$ , except in the vicinity of the upper and lower plates, and at any time most of the gas in the convection cell is at a temperature close to  $T_{\text{av}}$ . Gas which is in contact with the lower plate of the cell is heated to a temperature  $T_{\text{av}} + \Delta T$  (where  $0 \leq \Delta T \leq \Delta T_h/2$ ), and joins a plume of rising gas. The plumes are mixed by the turbulence in the interior of the cell. The plumes form fronts and later tendrils of approximately homogenous gas, which remain at a temperature close to the temperature that they had upon separation from the top or bottom plate until the last stage of the mixing process. In the final stage of mixing a tendril formed by the plume mixes rapidly with gas from the interior of the cell, which is at a temperature close to  $T_{\text{av}}$ . The final stage of mixing occurs very rapidly, on the Kolmogorov timescale,  $\tau_K = \sqrt{\nu/\epsilon}$ . The time  $\tau_{\text{mix}}$  between a plume separating and the gas surrounding a given aerosol particle becoming mixed is highly variable:  $\tau_h \gg \tau_{\text{mix}} \gg \tau_K$ , where  $\tau_h = (h^2/\epsilon)^{1/3}$  is the turnover time of the largest eddies. The equilibration timescale  $\tau_{\text{eq}}$  will be assumed to lie between the timescales describing the flow:  $\tau_{\text{mix}} \gg \tau_{\text{eq}} \gg \tau_K$ .

The consequence of this picture is that droplets in a rising plume are at a temperature,  $T_{\text{av}} + \Delta T$ , and while the plume forms they equilibrate to a smaller radius: to leading order

$$a_- = a_0 - \Lambda \Delta T \quad (1)$$

where  $\Lambda$  is a constant discussed in [11]. The gas in the plume rises, without cooling due to heat exchange, until it reaches the interior of the cell. After a timescale  $\tau_{\text{mix}}$ , the gas in the plume starts to mix with the gas in the interior. This mixing happens on a timescale which is short compared to the phase equilibration time, so that droplets of size  $a_-$  are mixed with



**Figure 1.** Schematic plot of the temperature of a convected aerosol particle. The time  $\tau_{\text{mix}}$  between contacts with the upper plate may be as short as  $\tau_K$ . There may be a very large number of contacts with the upper plate before the particle reaches the lower plate, which requires a much longer time, of order  $\tau_h$ .

the droplets in the bulk, which are of size  $a_0$ . Similarly, plumes of cold gas which form on the upper plate at a temperature  $T_{\text{av}} - \Delta T$  inject larger droplets, of radius  $a_+ = a_0 + \Lambda \Delta T$ . The final stage of this mixing happens on a timescale of the Kolmogorov time  $\tau_K$ , which is small compared to the time  $\tau_{\text{eq}}$  required for aerosol droplets to come into equilibrium. It follows that while the temperature fluctuations associated with the plume are dissipated, fluctuations in the droplet size remain ‘frozen in’, resulting in a broadening of the droplet size distribution.

The change in droplet radius arises from the abrupt change in temperature at the hot and cold plates of the convection cell. Consider the evolution of a droplet which spends some time close to the top of the cell and which has repeated encounters with the cold plate. As its temperature rises and falls, we might expect that the radius of the droplet would increase and decrease, with a minimal net effect. However, there can be a marked asymmetry between the timescales of heating and cooling. In a convection cell the gas is cooled relatively slowly by the upper plate, before being warmed rapidly by turbulent mixing when a plume falls into the interior. If the cooling occurs slowly compared to  $\tau_{\text{eq}}$ , droplets grow by condensation. If the warming due to mixing is on a timescale  $\tau_K \ll \tau_{\text{eq}}$ , the larger droplets do not evaporate to their original size. The asymmetry between the cooling and heating processes allows the droplets to have a systematic growth, rather than a cyclic fluctuation of size.

The production of rain from clouds depends upon droplets reaching a size which is significantly larger than their original size. In the context of the Rayleigh-Bénard model, this would require a droplet to undergo repeated encounters with the cold plate. The time  $\tau_{\text{mix}}$  that an aerosol particle spends in a plume before it is mixed is highly variable and it will usually be very short compared to the integral time  $\tau_h$ . It follows that some droplets may experience many interactions with the upper plate in rapid succession, as illustrated in figure 1.

## APPLICATION TO RAINFALL

The talk will describe how this non-collisional model for increasing the dispersion of droplet sizes in a Rayleigh-Bénard cell can be applied to understanding the rapid onset of rainfall from cumulus clouds. It will be argued that this mechanism leads to a resolution of the droplet growth bottleneck problem in cloud physics. It has been argued above that the dominant mechanism for creating larger droplets is that droplets grow by condensation at the cold upper surface of a cloud, but that the increased size is frozen in when a falling plume of cold air is mixed rapidly in the interior of the cloud. The talk is based upon a recent publication, [11].

## References

- [1] B. J. Mason, *The Physics of Clouds*, University Press, Oxford, (1957).
- [2] H. R. Pruppacher and J. D. Klett, *Microphysics of Clouds and Precipitation*, 2nd ed., Kluwer, Dordrecht, (1997).
- [3] R. A. Shaw, *Ann. Rev. Fluid Mech.* **35**, 183-227, (2003).
- [4] A. B. Kostinski and R. A. Shaw, *Bull. Am. Met. Soc.*, **86**, 235-44, (2005).
- [5] J-L. Brenguier and L. Chaumat, *J. Atmos. Sci.* **58**, 628, (2001).
- [6] A. Celani, G. Falkovich, A. Mazzino, A., and A. Seminara, *Europhys. Lett.*, **70**, 775, (2005).
- [7] R. S. R. Sidin, R. H. A. IJzermans and M. W. Reeks, *Phys. Fluids* **21**, 106603, (2009).
- [8] A. S. Lanotte, A. Seminara, and F. Toschi, *J. Atmos. Sci.*, **66**, 1685, (2009).
- [9] B. Kumar, J. Schumacher and R. A. Shaw, *Theor. Comput. Fluid Dyn.*, **27**, 361-76, (2013).
- [10] G. Ahlers, S. Grossmann and D. Lohse, *Rev. Mod. Phys.*, **81**, 503, (2009)
- [11] M. Wilkinson, *Europhys. Lett.*, **108**, 49001, (2014).