GENERALIZED DIAGNOSTIC SCALING FOR HIGH-ORDER MOMENTS IN TURBULENT BOUNDARY LAYERS

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Abstract The present work builds upon the diagnostic plot for the streamwise turbulence intensity [Alfredsson & Örlü, 2010] and generalises it for higher-order (even and odd) moments providing a general description of the probability density distribution of streamwise velocity fluctuations. Turbulent boundary layers (up to a friction Reynolds number of 20'000) are employed and demonstrate its feasibility to scale data throughout the overlap and outer region.

BACKGROUND AND MOTIVATION

The recent emergence of reasonably well-resolved data from high Reynolds number (Re) pipe and turbulent boundary layer (TBL) flows has challenged the classical understanding of wall turbulence. In particular, the failure of inner-scaling (henceforth denoted through the superscript '$+$') for the streamwise velocity rms profile (Fig. 1a) has avalanched a number of experimental campaigns to provide well-resolved high-Re data both in pipe and TBL flows (see e.g. Ref. [6] and references therein). While the debate regarding the scaling of the inner peak of the variance profile (at least in TBL flows) has come close to a settlement [6], the emergence of a second peak (cf. Fig. 1a) as well as Refs. [5, 12] among others) continues to stir the minds: relating its presence either to structural changes in the flow [9], a predictable feature inherent already in lower Re data [3] or simply to measurement insufficiencies.

The availability of high Re data has, on the other hand, compiled compelling empirical evidence for the existence of a logarithmic mean velocity profile [7]. At the same time, the data at hand has resurrected the actuality of the logarithmic law for the streamwise variance profile, as predicted by Townsend [11] based on the attached-eddy hypothesis. Recent efforts have probed its validity for the variance, but also higher-order even moments [8, 13] as also depicted in Fig. 1b for the 2nd to 10th central moment. As apparent in that figure, a logarithmic behaviour is visible for the a wide part of the overlap region for all presented even moments following $\langle u^{+2p}\rangle^{1/p} = D_p(Re_\tau) - A_p \ln y^+$, where D_p and A_p are flow case, Re and moment-dependent parameters, where A_p is expected to asymptote to a universal value at large Re [13].

An alternative scaling behaviour has emerged from the diagnostic plot [1], where the turbulence intensity is plotted against the outer-scaled mean velocity (U/U_∞) rather than wall distance. The diagnostic plot has shown promising success to collapse TBL data (but also pipe and channel flows) covering a wide Re range throughout the overlap and outer layer [4] as also apparent from Fig. 1c, where the data from Fig. 1a has been brought to collapse; linearly dependent on U/U_∞. Hence, with a measured or modelled mean velocity profile, the streamwise variance profile of pipe, channel and TBL flows could be described [4]. Based on this success, the present efforts aim at extending the description to higher-order (both even and odd) central moments.

RESULTS AND OUTLOOK

The probability density function (PDF) of the streamwise velocity fluctuations (scaled by U) for the three Re-cases shown in Fig. 1b) are as well plotted in diagnostic form in Fig. 2a. Since the contour lines of the PDFs, irrespective of Re, collapse both near the median and in the tails of the PDF, also higher-order moments should scale in diagnostic form. A generalised form of higher order moments in diagnostic scaling can be expressed as $|\langle u^n\rangle|^{1/n}/U$ as function of U/U_∞ and is shown in Fig. 2b for odd moments ($n = 3, 9, 17$) and in Fig. 2c for even moments ($n = 4, 10, 18$). These are the same 29 data sets as shown in Fig. 1a and as can be seen they collapse irrespective of Reynolds number throughout the boundary layer except in the region $0.4 \lesssim U/U_\infty \lesssim 0.6$. This region corresponds approximately to $15 \lesssim y^+ \lesssim 100$.

Indeed, the shown collapse of data over a wide Re-range throughout the boundary layer, with the exception of the intermediate layer, generalises the diagnostic scaling to higher-order even and odd moments. However the clear spread of the data in the region $0.4 \lesssim U/U_\infty \lesssim 0.6$ shows that this region does not exhibit self-similarity in the fluctuations. One may hypothesise that this is due to an influence on the fluctuations both from the wall itself and from the outer region and that this mixed influence prohibits self-similarity of the fluctuations. Further away from the wall the restrictive influence of the wall (no-slip and non-permeability) becomes negligible and the outer region becomes self-similar. In close wall proximity, the viscous sublayer acts more or less as a lubrication layer which is forced by the outer flow, resulting in a near log-normal scaling of the velocity fluctuations [2], thereby the outer flow is forcing a more or less passive layer. The implications of the found generalised diagnostic scaling as well as its relation to the generalised logarithmic law will be expanded on in the final presentation.

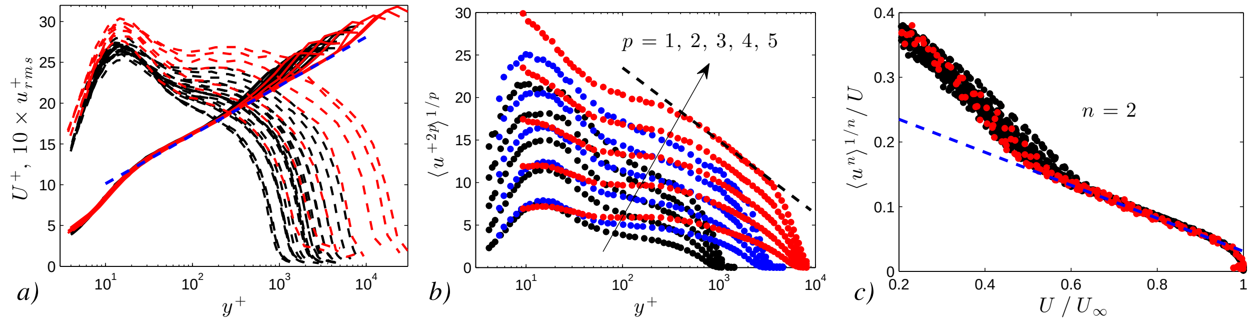

Figure 1. *a)* Inner-scaled streamwise mean (solid line) and rms (dashed line) profiles for 22 (black) and 7 (red) ZPG TBL experiments covering a range of $Re_\tau = 800 - 6000$ [10] and $1400 - 20000$ [12], respectively. Blue dashed line indicates the logarithmic law with constants given in Ref. [12]. *b)* Generalized logarithmic law for the $2p$-order moments for $Re_\tau = 850$ (black), 2400 (blue) and 5600 (red). Dashed lines indicates the logarithmic behaviour for the 10th moment at the highest Re. *c)* Extended diagnostic plot for the entire data set shown under *a)*. Dashed line corresponds to the linear fit with constants given in Ref. [4].

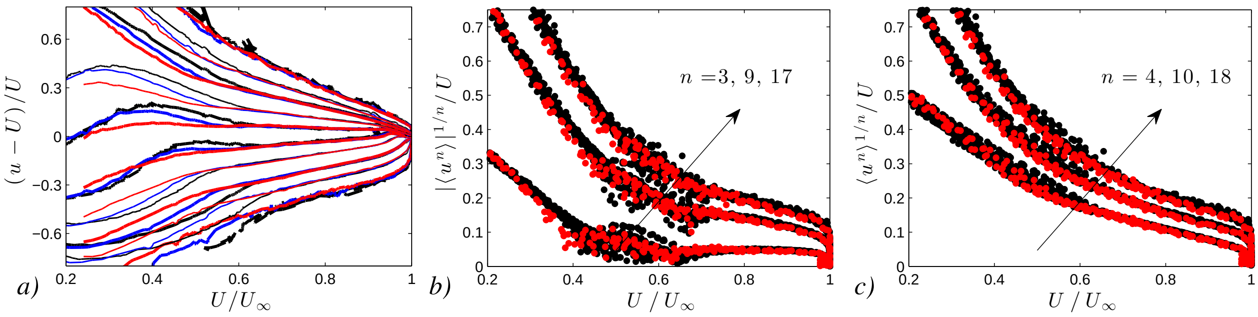

Figure 2. *a)* PDF map in diagnostic scaling for the same profiles shown in Fig. 1*b*. Contour levels correspond to **0.01**, 1, **10**, 50, and **90**% of the maximum PDF value of each Re case. *b)* Generalized diagnostic plot for odd moments for the entire data set shown under Fig. 1*a*. Note that in case of odd moments, the absolute value had to be considered before applying the root operator. *c)* Generalized diagnostic plot for even moments for the entire data set shown in Fig. 1*a*.

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