TURBULENT ENTRAINMENT IN JETS AND PLUMES

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Abstract We perform direct simulation of a statistically steady jets and plume and present the value of the entrainment coefficient decomposed into 1) turbulence production; 2) buoyancy effects; and 3) deviations from self-similarity. We explain theoretically how the two cases are linked and present a generalisation valid for forced and lazy plumes.

The value of the entrainment coefficient α for jets and plumes has been the subject of much debate since the inception of the entrainment hypothesis in the fifties [4], with typical top-hat values in the range $0.064 < \alpha < 0.079$ in pure jets and $0.10 < \alpha < 0.16$ in pure plumes [1]. Notably, the value of α is significantly larger for plumes than for jets and we study the energetics of entrainment to explain this behaviour.

For an axisymmetric plume, the entrainment flux is given by $q = -2r\overline{u}|_\infty$ where $\overline{u}$ is the radial velocity. This flux is usually parameterised as $q = 2\alpha M^{1/2}$ where $M = \int_0^\infty \overline{w}^2 r\mathrm{d}r$ is the integral of the streamwise momentum flux. By making use of the equation for mean kinetic energy it is possible to decompose α into several contributions [3]. Starting from recent work on unsteady jets [2], it follows that α can be decomposed as

$$\alpha = \underbrace{-\frac{\delta}{2\gamma}}_{\alpha_{\text{prod}}} + \underbrace{\left(\frac{1}{\beta} - \frac{\theta}{\gamma}\right)\text{Ri}}_{\alpha_{\text{Ri}}} + \underbrace{\frac{Q}{M^{1/2}}\frac{\mathrm{d}}{\mathrm{d}z}\left(\log\frac{\gamma^{1/2}}{\beta}\right)}_{\alpha_{\text{sim}}} \qquad (1)$$

where Q and F are the volume flux and integral buoyancy flux, respectively, and $\text{Ri} = FQ^2/\theta M^{5/2}$ is the plume Richardson number. The coefficients β, γ, and θ represent the profile constants for the momentum, energy and buoyancy flux, respectively. For Gaussian profiles, they take the value $\beta = 1$, $\gamma = 4/3$ and $\theta = 1$.

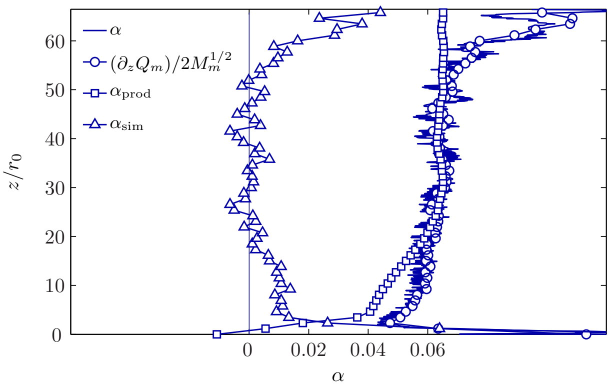

Figure 1. Decomposed turbulent entrainment coefficient in a steady jet at Re $= 6815$.

Equation (1) decomposes α into contributions from 1) turbulence production (α_{prod}); 2) buoyancy effects (α_{Ri}); and 3) changes in profile shape which are generally restricted to the near-field. Shown in figure 1 is the decomposed entrainment coefficient for a steady jet at Re $= 6815$. The data was obtained using direct numerical simulation, and a detailed description of the code and validation can be found in [2]. First and foremost, the figure demonstrates that (1) is consistent with a direct definition stemming from the continuity equation $\alpha = (2M^{1/2})^{-1}\mathrm{d}Q/dz$. Furthermore, the figure shows that α_{prod} is the only significant contribution in the far field. Note that α_{sim} is nonzero in the near field only, implying that the velocity profile changes until it finds its self-similar shape. The values for $z/r_0 > 50$ are affected by the outflow boundary and can be ignored. In the presentation, we will show the decomposed entrainment coefficient for a statistically steady turbulent plume and explain how these two cases can be linked theoretically. Finally, we discuss how the theory can be extended to forced plumes and lazy plumes.

References

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