EMERGENCE OF OBLIQUE TS MODE DUE TO LONGITUDINAL WALL OSCILLATION IN 2D CHANNEL FLOW

Takashi Atobe
Japan Aerospace Exploration Agency, Tokyo, Japan

Abstract Stabilizing effects of wall oscillation on two dimensional (2D) Tollmein-Schlichting (TS) waves and oblique TS waves developing in 2D channel flow are numerically investigated using the Floquet method. It was shown from the previous study[1] that the wall oscillation mostly has stabilizing effect and then TS mode is occasionally more stable than the oblique mode. In the present study the characteristics of the stability are investigated in detail. Then it is cleared that the exchange of the most unstable mode form the 2D TS to the oblique TS can occur at a certain value of Reynolds number.

INTRODUCTION

The drag reduction can be classified into two ways, the passive control and the active one. The oscillating wall is an example of the latter one, and Quadrio and Rico numerically showed about 44% drag reduction[2]. It can be thought in this case that the drag reduction is caused by the change of the vortex structures or the velocity profiles near the wall. It means that the wall oscillation strongly affects the stability of base flow. Thus the present study focuses on the linear stability of the flow and estimates the effects of the wall oscillation by using the amplification ratio of the small disturbances. Because of the periodicity of the flow, the Floquet method is employed for the stability analysis.

MODEL FLOW

The schematic view of the model flow is shown in Fig.1. The parameters governing this system are the amplitude of wall oscillation *Uw* and its frequency Ω, and the Reynolds number *Re* which is determined by the maximum velocity of the mean flow and the half width of the walls. In this model, the velocity is thought as a linear combination of 2D Poiseuille flow and the Stokes layer.

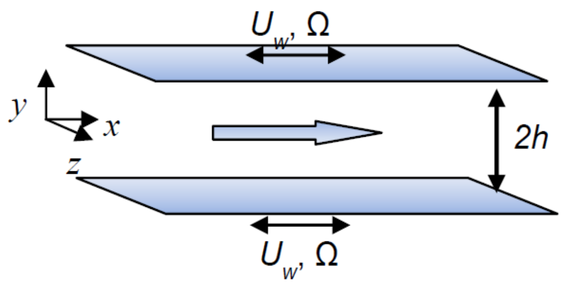


Figure 1. The schematic view of the model

The time dependent Orr-Sommerfeld equation is used in the stability analysis and given in eq.(1). Hear *x*, *y*, *z* are the streamwise, wall-normal, spanwise direction, and α, γ are the wave number for *x* and *z* directions. Also *D* is differentiation operator in *y* direction. The velocity *U* as a linear combination is given in eq.(2).

$$[(\frac{\partial}{\partial t}+i\alpha U(y,t))(D^2-\alpha^2-\gamma^2)-i\alpha D^2U(y,t)]\hat{v}(y,t)=\frac{1}{R}(D^2-\alpha^2-\gamma^2)^2\hat{v}(y,t)\,. \quad (1)$$

$$U(y,t)=1-y^2+U_w\,\mathrm{Re}[\frac{\cosh(\kappa y)}{\cosh(\kappa)}]\exp(i\Omega t)\,. \quad (2)$$

Here $\kappa=\sqrt{\Omega/2\nu}$, and *i* is the imaginary unit. With the expansion in *y* direction by the Chebyshev collocation method, eq(1) can be rewrote as

$$\frac{d}{dt}F(t)=G(t)F(t)\cdot \quad (3)$$

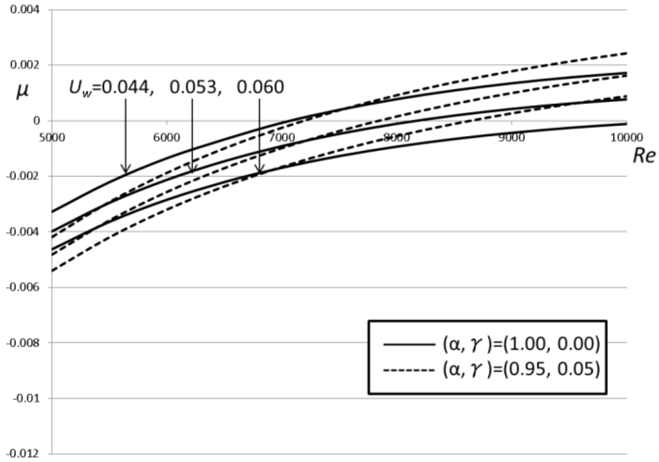
Then the Floquet exponent μ as a criterion of the stability is determined as the eigenvalue of matrix *Q* which is defined by the follows. The detail is shown in Ref[2].

$$Q=\frac{1}{T}\ln F\cdot \quad (4)$$

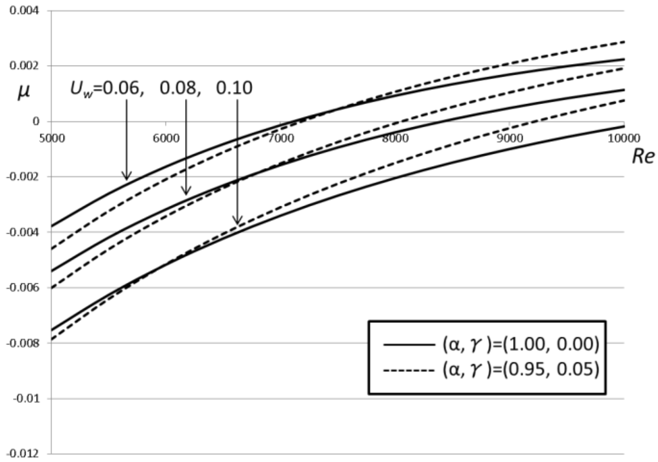
COMPUTATIONAL RESULTS

It was cleared from the previous study that the wall oscillation can stabilize both of the TS and the Oblique TS mode[1]. To understand this characteristics in more detail, *Re* dependence of the stabilizing effect is investigated. Figure 2 shows the variation of the Floquet exponents with *Re*. Since the critical *Re* of the channel flow on the present definition is 5772, all the case shown in these figures are stabilized. It is seen that the stabilizing effect on the oblique TS mode becomes weaker for large *Re* region. As the result, the exchange of the most unstable mode from TS to the oblique TS occurs at a certain *Re* value. If this exchange occurs at a negative μ value, the oblique TS mode is more unstable and may appear fist than the TS mode against the Squire's theorem.

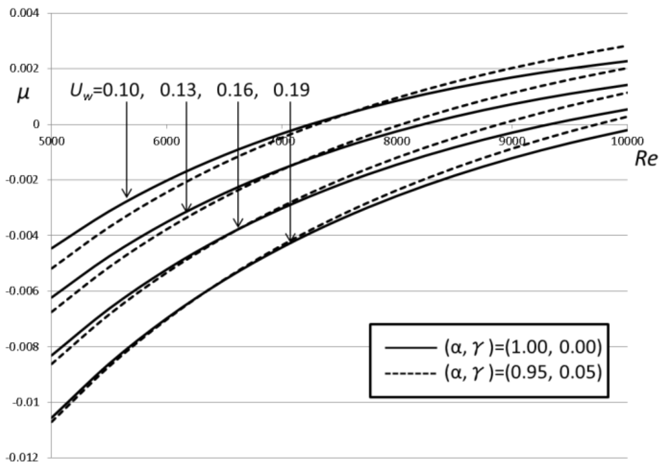
It is also seen that the more the amplitude of the wall oscillation *Uw* increases, the more the Floquet exponents μ decreases. Furthermore, *Re* occurring the exchange of the most unstable mode changes smaller. This tendency is same for all cases in Fig.2. For the emergence of the oblique TS wave, the important thing is that the sign of the Floquet exponent what the exchange occurs is positive or negative. If the exchange occurs with negative value of the Floquet exponent, the oblique TS mode can appear fist. Figure 3 shows the variation of *Re* occurring the exchange with *Uw* for Ω=0.1, 0.15, and 0.2. The solid lines correspond to the positive value of the Floquet exponent. Thus this figure suggests that the oblique TS mode can't appear for less than *Re*≃7,300.



(a) Ω=0.1.



(b) Ω=0.15.



(c) Ω=0.2.

Figure 2. Variation of the Floquet exponent with *Re.*

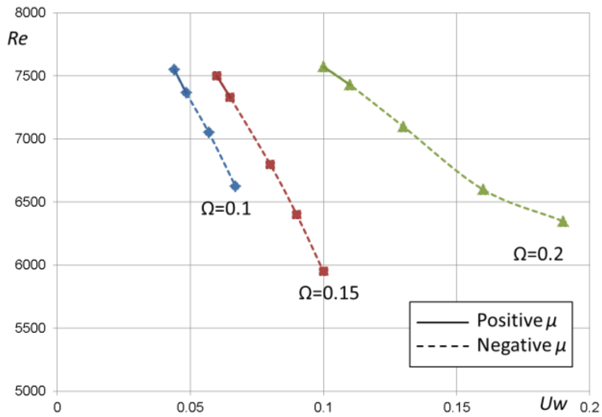


Figure 3. Variation of *Re* with *Uw*. This Re corresponds to the value occurring the exchange of the most unstable mode form the TS to the oblique TS.

CONCLUSIONS

Stability of 2D channel flow with the longitudinal wall oscillation is analytically investigated by using the Floquet theorem. In this approach, the time dependent Orr-Sommerfeld equation is employed with the Chebyshev collocation method. It is cleared from the present study that the wall oscillation has stabilizing effect and this effect is larger when the amplitude of the wall oscillation is large. It is also found that although this stabilizing effect is seen for the oblique TS mode too, this effect is smaller for large *Re* region. This implies that the oblique TS mode can appear fist than 2D TS mode against the Squire's theorem. And this phenomena may occur at *Re* more than about 7,300.

References

[1] M. Quadrio and R. Ricco. Critical Assessment of Turbulent Drag Reduction Through Spanwise Wall Oscillation *J. Fluid Mech.* **521**:251-271, 2004.

[2] T. Atobe, Primary mode changes due to longitudinal wall oscillation in two dimensional channel flow. *Fluid Dyn. Res.* **46** 025502, 2014.