Turbulent pipe flow: Statistics, \(Re\)-dependence, structures and similarities with other wall flows

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1 Overview

In wall-bounded flows, there are three simple geometrical configurations which are referred to as the canonical cases: the spatially evolving boundary layer, the channel and the pipe. Only recently, computer power has grown sufficiently large to attempt fully resolved numerical solutions of all relevant turbulent scales. While the first of these two flow configurations have been studied extensively using direct numerical simulation (DNS) in the past years, there are only a limited number of numerical studies carried out for pipe flows.

Considerable interest in high \(Re\) pipe flows stems from the still many open question relating to the scaling of turbulent statistics as reviewed by Marusic et al. (2010)\textsuperscript{[1]}. Essentially, one would expect that various simulations and experiments on pipe flows, once the axial extent is chosen sufficiently large, agree well with each other. However, in spatially developing turbulent boundary layers, a recent comparison between a number of available simulation data sets by Schlatter & Örlü (2010)\textsuperscript{[2]} has shown a surprisingly large spread of the data even for basic quantities such as the shape factor of friction coefficient, indicating that not even the mean profiles agreed between the various DNS. It is therefore interesting to see whether the expected close agreement of DNS data in pipe can be confirmed based on the available literature data, and to what extent these data agree with corresponding simulations in channel and boundary-layer geometries. To this end, we have performed well resolved DNS of turbulent pipe flow in sufficiently long pipe (25\(R\)) at four different Reynolds numbers up to \(Re\_τ = 1000\). Here, the Reynolds number is based on the pipe radius \(R\) and friction velocity \(u_τ\). These simulations have been performed using the spectral element method (SEM) and the grid spacing, measured in wall units, is set such that \(Δr^+\_{max} ≤ 5\) with four and fourteen grid points placed below \(Δr^+ = 1\) and 10 (from the wall), respectively, \(ΔRθ^+_{max} ≤ 5\) and \(Δz^+_{max} ≤ 10\).

The details of the computational meshes at the various Reynolds numbers are presented in table 1. These data sets will be available in addition to a well-resolved DNS on turbulent pipe flow by Bendiks Jan Boersma at \(Re\_τ = 1700\).

An extensive investigation of the similarities and differences between the canonical wall-bounded flows is yet to addressed. For example, it is interesting to note that the maximum in the inner peak of axial turbulence intensity is similar between channels and boundary layers while lower in pipes whereas the fluctuation wall shear stress is similar between pipe and channel flows and is lower than that in boundary layers. Is it that internal flows are similar at the wall while from the buffer layer, and towards the wake, channels and boundary layers have more similarities, what is the influence of the pipe geometry, and where does the discrepancies in pressure between the three flow arise from. Other questions relate to: the presence of large-scale structures and how much are these structures similar/different between the three flow cases, the energy fluxes in physical space and space of scales in pipe and boundary layers. Two-point correlations, spectra and conditional statistics in addition to second-order structure functions, extended to boundary layer and pipe flows, will be useful in this context.

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Table 1: Details on the present turbulent pipe flows simulations.

<table>
<thead>
<tr>
<th>$Re_b$</th>
<th># of elements</th>
<th># grid points</th>
<th>$\Delta r^+$</th>
<th>$\Delta R\theta^+$</th>
<th>$\Delta z^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5,300</td>
<td>36,480</td>
<td>$18.67 \times 10^6$</td>
<td>(0.14, 4.44)</td>
<td>(1.51, 4.93)</td>
<td>(3.03, 9.91)</td>
</tr>
<tr>
<td>11,700</td>
<td>237,120</td>
<td>$121.4 \times 10^6$</td>
<td>(0.16, 4.70)</td>
<td>(1.49, 4.93)</td>
<td>(3.03, 9.91)</td>
</tr>
<tr>
<td>19,000</td>
<td>853,632</td>
<td>$437.0 \times 10^6$</td>
<td>(0.15, 4.49)</td>
<td>(1.45, 4.75)</td>
<td>(3.06, 9.99)</td>
</tr>
<tr>
<td>37,700</td>
<td>1,264,032</td>
<td>$2.184 \times 10^9$</td>
<td>(0.15, 5.12)</td>
<td>(0.98, 4.87)</td>
<td>(2.01, 9.98)</td>
</tr>
</tbody>
</table>

2 Questions to be addressed/Schedule

During the First Multiflow Summer Workshop we would like to address the following issues by analysing existing databases, both from KTH, Madrid and other places.

- **Pipe-flow statistics up to $Re_\tau = 1000$**. Assessment of turbulent pipe flow DNS data by means of comparisons with experimental data and previous/on-going numerical simulations on pipe flows from different resources; e.g. Netherlands, United States, Australia, South Korea.

- **Quantification of confidence intervals/uncertainties for DNS data**. Discrepancies, whether large or small, are often observed between various DNS data sets. Some of these differences arise from the numerical schemes, average time, starting time for averaging and others due to unphysical phenomena. A method to quantify confidence intervals in DNS data is thus desirable as it will give a range over which the data is considered reliable. Confidence intervals could be estimated for statistical quantities such as the maximum in the inner peak axial rms, and various moments in the wake region.

- **Similarities and differences with other wall-bounded flows**. Pipe is one the three canonical wall bounded flows, together with turbulent channel flow and boundary layer flow. Which statistics and turbulence structures are similar among the different types of wall-bounded flows and which are different? What are potential explanations to the observed discrepancies? The pressure is the variable which differs the most between pipes, channels and boundary layers, leading to significantly higher mean pressure in the outer region of channels and boundary layers, potentially linked to a stronger wake region. In the inner region, interesting discrepancies can be observed between the canonical flows. The variation with Reynolds number of the inner peak of axial velocity fluctuation is similar between channel and boundary layer but lower for the pipes whereas the inner peak of the pressure fluctuation show negligible difference between pipe and channel flow but is clearly lower than that for the boundary layer caused by the different influences of large-scale motions in the outer layer.

- **Long structures**. There is experimental evidence that very long structures, longer than $10D$, exist in turbulent pipe flow, see for instance Kim & Adrian (1999) [3] and Monty et al. (2007) [4]. Two-point correlations, spectra and conditional statistics can verify whether such long structures can indeed be found in the DNS data. Furthermore, their energy content can be established. Are the large-scale structures different in pipe, channel and boundary layer flows as indicated previously and what are the implications?

- **Interaction between large- and small-scale structures and their importance for momentum transfer**. New statistical tools can clarify how the near-wall structures and large structures interact or modulate each other, as for instance shown and used by Marusic et al. (2010) [5]. In addition, the energy flow among different scales bears important implications for the dynamics and regeneration of wall turbulence. Cimarelli & De Angelis (2011) [6] and more recently Cimarelli et al. (2013) [7] have employed the second-order structure function to study the path of energy in the physical space as well as the space of scales of turbulent channel flow. This approach yields a more complete description of the path of the kinetic energy and the results can have important consequences for LES of wall-bounded flows and wall modeling. The extension of the
above method to boundary layers and pipes would give a useful insight into the energy fluxes in the latter canonical wall-bounded flows. However, this requires a new set of equation for the second-order structure function that is yet more complex due to the spatial development of boundary layers and curved nature of pipes.

2.1 Available databases and models

The data analysis is mainly based on data bases obtained by means of direct numerical simulation during the past year. The pipe flow simulations are conducted using the spectral element code nek5000 [8] with indisputable resolution and in a sufficiently long pipe of length \(25R\). The channel and boundary layer data, on the other hand, are obtained from the well documented spectral solver simson. All simulation data is well resolved in agreement with recent guidelines, and all data will be available during the workshop to all participants.

3 Required Resources

Linux workstations and computational resources for post-processing and visualisation. However, computational/storage resources in Sweden will be available during the program. KTH Mechanics can supplement the stipend for the summer programme for its participants, in particular travelling costs. Simple housing for the participants and limited per-diem would be appreciated.

4 Relation to other proposals

During the summer school we intend to collaborate with Ricardo Vinuesa Motilva from Illinois Institute of Technology/Polytechnic University of Madrid and Andrea Cimarelli from University of Bologna. Ricardo Vinuesa is currently working on turbulent duct flows with different aspect ratios. We already have contact and ongoing work with Ricardo who is using the same numerical method as our group; the DNS code nek5000 and the collaboration with him would revolve around feature extraction of turbulent structures in turbulent duct flows. The joint work with Andrea Cimarelli, on the other hand, would be on energy fluxes in the space of scales and physical space of wall-bounded turbulent flows; in particular for turbulent boundary layer and pipe flows.

5 Brief description of the applicants

Philipp Schlatter and Geert Brethouwer are faculty members at KTH Mechanics (Sweden). George El Khoury joined KTH Mechanics in 2011 as Postdoc, after his PhD studies at NTNU Trondheim (Norway). Bendiks Jan Boersma is a full professor in Energy Technology at TU Delft (The Netherlands), and he would like to participate for a few days.

References


Figure 1: Isosurfaces of negative $\lambda_2$ following Jeong & Hussain (2010) [12] coloured by axial velocity $u_z/U_b$ for $Re_\tau = 1000$.

Figure 2: (left) Profiles of mean axial velocity $U_z^+$ at $Re_\tau = 180, 360, 550$ and $1000$ in inner scaling. (right) $Re_\tau \approx 1000$. Pipe: , present DNS; channel: , Lenaers et al. (2012) [9]; boundary layer: , Schlatter & Örlü (2010) [2]. $U_z^+ = (1-r)^+$ and $U_z^+ = \kappa^{-1}\ln(1-r)^+ + B$ with $\kappa = 0.41$ and $B = 5.2$ are given as dashed lines.

Figure 3: Premultiplied energy spectra of the axial velocity component $u_z$ for $Re_\tau = 550$. (left) 1D-spectra $k \cdot \phi(u_z u_z)/u_z^2$. (right) 2D-spectra $k_z k_\theta \cdot \phi(u_z u_z)/u_z^2$ at $(1-r)^+ = 15$. 